

**Double magic nuclei for  $Z > 82$  and  $N > 126$** H. F. Zhang,<sup>1,2,\*</sup> Y. Gao,<sup>1,3</sup> N. Wang,<sup>4</sup> J. Q. Li,<sup>1,2</sup> E. G. Zhao,<sup>5</sup> and G. Royer<sup>6</sup><sup>1</sup>*School of Nuclear Science and Technology, Lanzhou University, Lanzhou 730000, People's Republic of China*<sup>2</sup>*Center of Theoretical Nuclear Physics, National Laboratory of Heavy-Ion Accelerator, Lanzhou 730000, People's Republic of China*<sup>3</sup>*School of Information Engineering, Hangzhou Dianzi University, Hangzhou 310018, People's Republic of China*<sup>4</sup>*Department of Physics, Guangxi Normal University, Guilin 541004, People's Republic of China*<sup>5</sup>*Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, People's Republic of China*<sup>6</sup>*Laboratoire Subatech, UMR: IN2P3/CNRS-Université-Ecole des Mines, Nantes 44, France*

(Received 14 October 2011; published 25 January 2012)

The “island of stability” of superheavy nuclei due to shell effects is explored and the  $\alpha$ -decay half-lives of these nuclei are predicted. The calculations of the binding energies within a new macroscopic-microscopic model (MMM) are performed and compared with the experimental data for heavy nuclei from Md to the  $Z = 118$  element. The agreement is excellent. The data confirm that the  $^{270}\text{Hs}$  is a deformed double submagic nucleus beyond  $^{208}\text{Pb}$ . The features of  $\alpha$ -decay energies and one-proton-separation energies from the MMM reveal that the next double magic nucleus after  $^{270}\text{Hs}$  should be  $^{298}114$ . The potential energy surfaces calculated within the constrained relativistic mean-field (CRMf) theory show that the  $^{270}\text{Hs}$  is a deformed double magic nucleus, but  $^{298}114$  is a spherical double magic nucleus. The  $\alpha$ -decay half-lives are determined using a generalized liquid drop model (GLDM) with the  $Q_\alpha$  from the MMM for Hs and  $Z = 114$  isotopes, respectively.

DOI: [10.1103/PhysRevC.85.014325](https://doi.org/10.1103/PhysRevC.85.014325)

PACS number(s): 27.90.+b, 23.60.+e, 21.10.Tg, 21.60.-n

**I. INTRODUCTION**

The existence of an “island of stability” of superheavy nuclei (SHN) is predicted in the remote corner of the nuclear chart around the superheavy elements 114 to 126 due to shell effects. The recent discovery of new elements with atomic numbers  $Z \geq 110$  has brought much excitement to the atomic and nuclear physics communities. The experimental efforts have been focused on the direct creation of superheavy elements in heavy-ion fusion reactions, leading to the production of elements up to proton number  $Z = 118$  up to now [1–7]. The half-life of the new synthesized isotope  $^{287}114$  (several seconds) is several times shorter than that of the previously observed heavier isotope  $^{289}114$  ( $T_\alpha \approx 20$  s), formed in the reaction  $^{48}\text{Ca} + ^{244}\text{Pu}$  [6,7]. Such a trend is expected to be associated with a decrease of the neutron number. The observed radioactive properties of the new nucleus  $^{287}114$ , together with the data obtained earlier for the isotope  $^{289}114$  and the products of its  $\alpha$  decay (namely, the isotopes  $^{283}\text{Cn}$  and  $^{285}\text{Cn}$ ) can be considered as experimental proof of the approach of the “island of stability” of superheavy elements around  $Z = 114$ .

Theoretically, it had been concluded that the existence of the heaviest nuclei with  $Z > 104$  was primarily determined by the shell effects in the 1960s [8–10]. These early calculations predicted that the nucleus with  $Z = 114$  and  $N = 184$  is the center of an island of long-lived SHN. Recently, the detailed spectroscopic studies were performed [11–13] for nuclei beyond fermium ( $Z = 100$ ), with the aim of understanding the underlying single-particle structure of superheavy elements. The microscopic models are, however, still uncertain when

extrapolating in  $Z$  and the mass number  $A$ . In particular, there is no consensus among theorists with regard to what should be the next doubly magic nucleus beyond  $^{208}\text{Pb}$  ( $Z = 82$ ,  $N = 126$ ). In the SHN, the density of single-particle energy levels is fairly large, so small energy shifts, such as those, for instance, due to poorly known parts of nuclear interaction, can be crucial for determining the shell stability. So an alternative choice is to develop the theoretical calculations taking into account all the experimental data to give reliable predictions for the properties of the SHN.

**II. DOUBLE MAGIC NUCLEI WITH  $Z > 82$  AND  $N > 126$  DUE TO MACROSCOPIC-MICROSCOPIC METHOD DATA**

Very recently, the macroscopic-microscopic method (MMM) was developed, the isospin and mass dependence of the model parameters being investigated with the Skyrme energy density function [14]. A very good improvement is that the macroscopic and microscopic parts in the proposed mass formula are closely connected to each other through the coefficient  $a_{\text{sym}}$  of the symmetry energy. Its main advantage is to provide reasonable mass extrapolations for exotic and heavy nuclei. The number of model parameters (13 independent parameters) is considerably reduced so as to be compared with the finite-range droplet model (FRDM), in which the number of parameters is about 40 [15]. The root-mean-square (rms) deviation with respect to 2149 measured nuclear masses is reduced to 0.441 MeV (the corresponding result with FRDM is 0.656 MeV), which should be one of the best results to date. Another most impressive improvement is that the rms deviation of  $\alpha$ -decay energies of 46 SHN is reduced to 0.263 MeV (the corresponding result with FRDM is 0.566 MeV), which allows us to give reliable predictions of

\* zhanghongfei@lzu.edu.cn

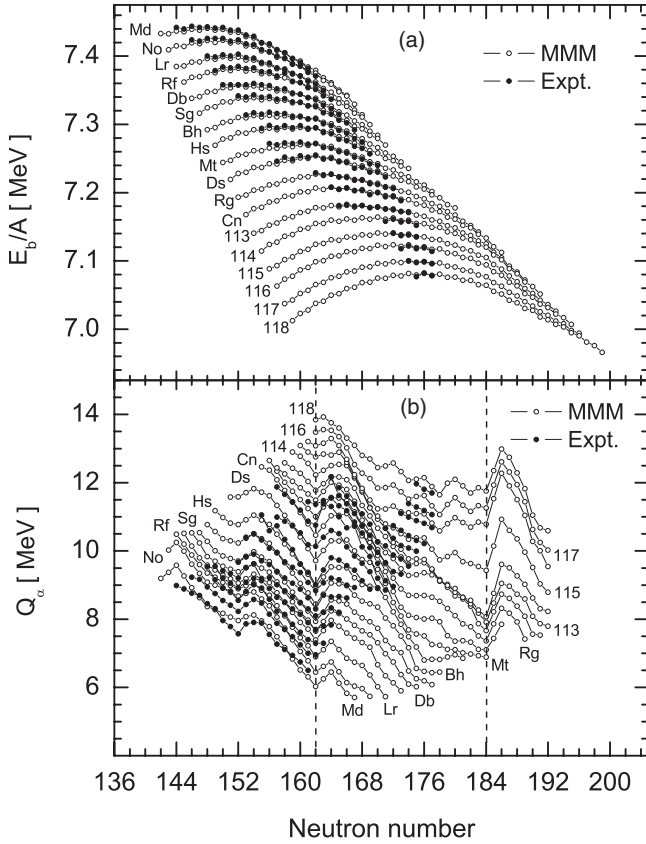


FIG. 1. Comparison of experimental binding energy  $E_b/A$  (upper panel) and  $\alpha$ -decay energies  $Q_\alpha$  (lower panel) with theoretical results. The vertical dotted lines indicate the magic-neutron numbers in the lower panel.

$\alpha$ -decay half-lives for SHN. It is meaningful to use the present data from the MMM to explore the features of SHN around the proposed “island of stability.”

First, we compared the binding energy of the MMM with the up-to-date nuclear data [16,17]. As shown in the upper panel of Fig. 1, the agreement between the MMM [14] calculations and the experimental results [16,17] is excellent for all the known nuclei from Md to  $Z = 118$  isotopes. This gives us full confidence to explore the  $\alpha$ -decay energies coming from the binding energies:  $Q = E_b^D + E_b^\alpha - E_b^P$ , where  $E_b^D$ ,  $E_b^\alpha$ , and  $E_b^P$  are the binding energies of the daughter nucleus,  $\alpha$  particle, and parent nucleus, respectively. The MMM  $\alpha$ -decay energies and the experimental values are shown in the lower panel of Fig. 1. The agreement between the two data is good. The lowest  $\alpha$ -decay energies are located at  $N = 162$  and  $184$ . If we check the results more carefully, one can observe that, from Md to Hs isotopes, the shell effect at neutron number  $N = 162$  is increasing, then decreasing, and nearly disappearing after the  $Z = 115$  isotopes. For  $N = 184$ , the shell effects increase from the Ds to  $Z = 114$  isotopes, then decrease until the isotope  $Z = 118$ . From Md to  $Z = 114$  isotopes,  $N = 162$  is the magic neutron number and from Ds to  $Z = 118$  isotopes  $N = 184$  is the magic neutron number. It is interesting to explore the proton magic number from the systematic properties of the SHN.

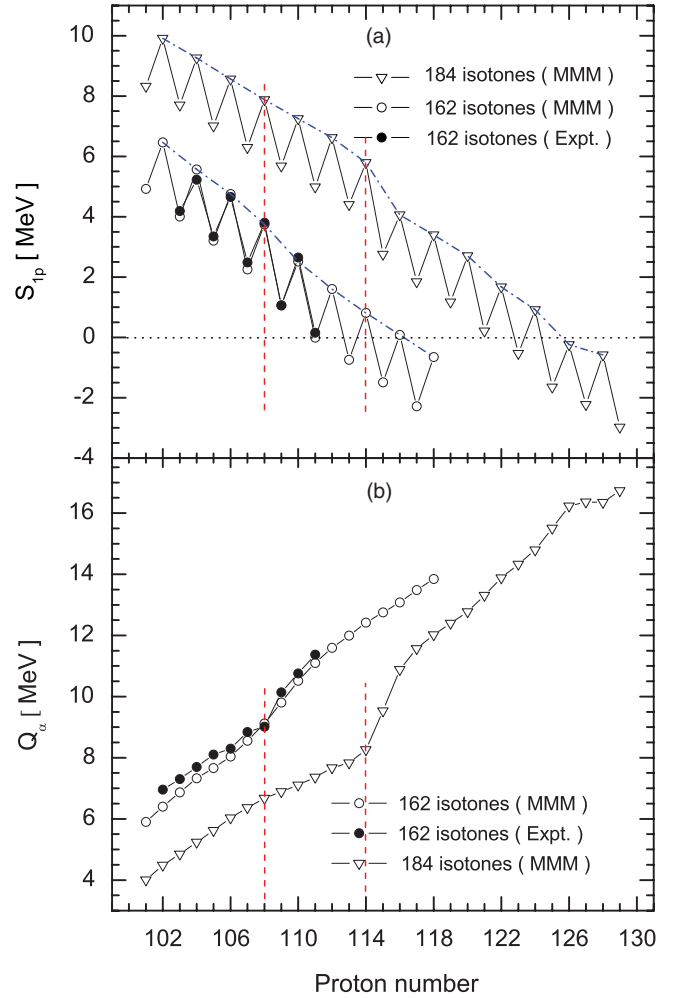


FIG. 2. (Color online) Comparison between the experimental one-proton-separation energies (upper panel) and  $\alpha$ -decay energies (lower panel) with the theoretical results for  $N = 162$  and  $N = 184$  isotones.

The one-proton-separation energy and  $\alpha$ -decay energy of the MMM [14] and experimental data [16,17] are shown in Fig. 2 for  $N = 162$  and  $N = 184$  isotones to find the proton magic number.  $S_{1p}$  generally decreases with increasing  $Z$  with obvious even-odd effect from the upper panel. With careful observation, it can be found that, at  $Z = 108$  and  $114$ , the values of  $S_{1p}$  are above the general trend, indicating that these nuclei are more stable. The results obtained by the MMM and the experimental data show clearly that the proton number  $Z = 108$  is a magic proton number for  $N = 162$  isotones and the calculated one-proton-separation energy of the MMM confirmed that  $Z = 114$  is a proton magic number for  $N = 184$  isotones. The  $\alpha$ -decay energies for  $N = 162$  and  $184$  isotones are shown in the lower panel of Fig. 2. Again we find the kinks of  $\alpha$ -decay-energy curves at  $Z = 108$  and  $114$ . The conclusions that both  $^{270}\text{Hs}$  and  $^{298}114$  are double magic nuclei after  $^{208}\text{Pb}$  are verified again and it is very interesting to study the ground-state deformations of the two nuclei.

In fact, most of superheavy nuclei found experimentally are known to be deformed. It is worthy to investigate the potential

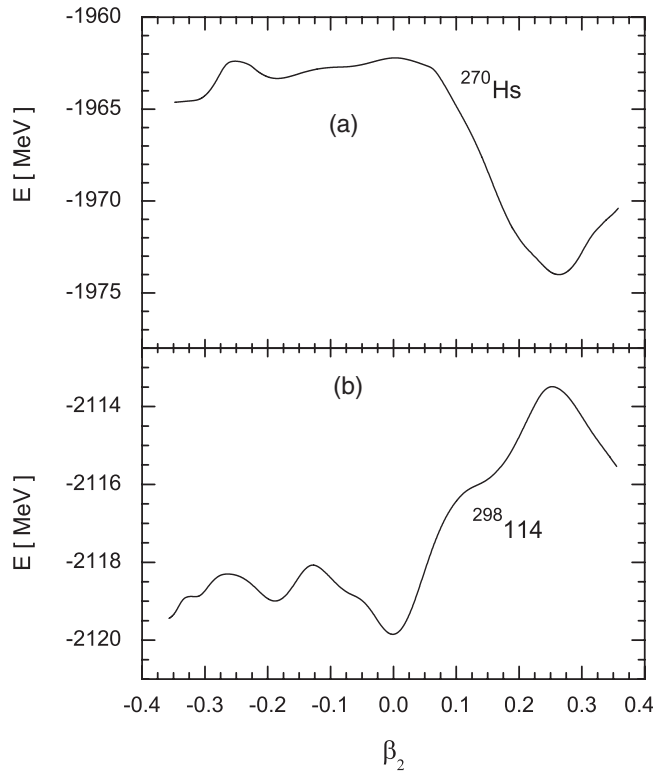


FIG. 3. Potential energy calculated in the constrained relativistic mean-field (CRMF) theory with effective interaction NL3 for  $^{270}\text{Hs}$  (upper panel) and  $^{298}114$  (lower panel).

energy surfaces in order to see the validity of the lowest equilibrium deformation. It is well known that the relativistic mean-field (RMF) calculation gives a good description of the structure of nuclei throughout the periodic table [18–20]. In this paper, the potential energy surfaces of possible double magic nuclei are obtained by using the constrained relativistic mean-field (CRMF) theory [21] and the pairing correlations are coped with via the Bardeen-Cooper-Schrieffer (BCS) approximation [22]. The quadrupole deformation parameter  $\beta_2$  is set to the expected deformation to obtain high accuracy and reduce the computing time. The potential energy surfaces have been calculated for  $^{270}\text{Hs}$  and  $^{298}114$  with the successful parameter set NL3 [23].

In Fig. 3, the potential energies of the nuclei  $^{270}\text{Hs}$  and  $^{298}114$  are presented versus the deformation. For nucleus  $^{298}114$ , there is a local spherical minimum ( $\beta_2 \sim 0$ ). For nucleus  $^{270}\text{Hs}$ , the spherical minimum has completely disappeared while a well-deformed local minimum appears at  $\beta_2 \sim 0.26$ . So we can draw the conclusion that the nucleus  $^{270}\text{Hs}$  is a deformed double submagic nucleus and  $^{298}114$  is a spherical double magic nucleus.

### III. $\alpha$ -DECAY HALF-LIVES OF NEWLY OBSERVED SUPERHEAVY NUCLEI AND SOME PREDICTIONS

The main decay mode of SHN is  $\alpha$  emission. Recently, the  $\alpha$ -decay half-lives have been calculated within a tunneling effect through a potential barrier determined by a generalized

liquid drop model (GLDM) [24,25] and the Wenzel-Kramers-Brillouin (WKB) approximation. The penetration probability is estimated by

$$P = \exp \left[ -\frac{2}{\hbar} \int_{R_{\text{in}}}^{R_{\text{out}}} \sqrt{2B(r)[E(r) - E_{\text{sphere}}]} dr \right], \quad (1)$$

where two approximations are used:  $R_{\text{in}} = R_d + R_\alpha$  and  $B(r) = \mu$ , where  $\mu$  is the reduced mass, and  $R_{\text{out}}$  is simply  $e^2 Z_d Z_\alpha / Q_\alpha$ .

The decay constant can be written as following associated with the concept that  $\alpha$  decay should be a preformed cluster emission process [26]:

$$\lambda = P_\alpha \nu_0 P, \quad (2)$$

where  $P_\alpha$  is the  $\alpha$  particle preformation factor and  $\nu_0$  is the assault frequency [27,28]. Then the half-life can be calculated by  $T_\alpha = \frac{\ln 2}{\lambda}$ . The extracted numerical preformation factors are parametrized in the form of an analytic formula [28] including all the known nuclear data before 2009. In the past year, important progress was made experimentally on the composition of SHN. The new  $Z = 117$  element was observed in fusion reactions between  $^{48}\text{Ca}$  and  $^{249}\text{Bk}$  by the Joint Institute for Nuclear Research in Dubna [5], which is the heaviest odd- $Z$  nucleus to date. The isotopes  $^{289}114$  and  $^{288}114$  were composed using the fusion-evaporation reaction  $^{244}\text{Pu}(^{48}\text{Ca}, 3-4n)^{288,289}114$  at the newly installed Transactinide Separator and Chemistry Apparatus (TASCA) by GSI [29], and the neutron-deficient isotope  $^{285}114$  was produced using  $^{48}\text{Ca}$  irradiation of  $^{244}\text{Pu}$  targets at a center-of-target beam energy of 256 MeV ( $E^* = 50$  MeV) at Berkeley [4]. It is interesting to calculate the properties of these nuclei to check the present GLDM as well as the new experimental data.

In the upper part of Table I, the experimental  $Q_\alpha^{\text{expt}}$  and half-lives  $T_\alpha^{\text{expt}}$  [5], the theoretical  $\alpha$ -decay half-lives calculated by the present GLDM and density-dependent M3Y (DDM3Y) interaction using the measured  $Q_\alpha^{\text{expt}}$  are also presented for  $^{293,294}117$  and their decay products. It is evident that our calculations coincide perfectly with the experimental  $\alpha$ -decay half-life for  $^{293}117$  and its decay products ( $^{289}115$  and  $^{285}113$ ) when the error encountered in measured  $Q_\alpha$  values. The values of  $\alpha$ -decay half-lives by DDM3Y are always smaller by about 5 times to one order of magnitude than experimental results for the three nuclei. It is noticeable for the daughter nucleus  $^{285}113$ , two different  $\alpha$ -particle energies 9.78 and 9.48 MeV are detected. The calculated half-life using the former value is 5.49 s, which is perfectly consistent with the experimental 5.5 s. But when the latter value 9.48 MeV is adopted for calculation of the half-life, the theoretical value will be five times that of the experimental data. If we select the value 9.78 MeV as the  $\alpha$ -particle energy of the daughter nucleus  $^{285}113$ , the agreement between our theoretical calculations by the GLDM and the experimental results is very good.

For the nucleus  $^{294}117$  and its decay products, the theoretical results are reasonably consistent with experimental data. For  $^{294}117$ ,  $^{286}113$ , and  $^{278}\text{Mt}$  the experimental values of half-time are about 5–10 times larger than theoretical calculations. But for the nuclei  $^{290}115$  and  $^{282}\text{Rg}$ , the deviations

TABLE I.  $\alpha$ -decay half-lives of the observed  $^{293,294}117$  [5] and  $^{288,289}114$  [29] decay chains and the theoretical results of the present GLDM and calculations [30] by the DDM3Y interaction using measured  $Q_\alpha$  in MeV.

Nucleus	$Q_\alpha^{\text{expt}}$	$T_\alpha^{\text{expt}}$	$T_\alpha^{\text{GLDM}}$	$T_\alpha^{\text{DDM3Y}}$	Nucleus	$Q_\alpha^{\text{expt}}$	$T_\alpha^{\text{expt}}$	$T_\alpha^{\text{GLDM}}$	$T_\alpha^{\text{DDM3Y}}$
$^{293}117$	11.18(8)	14 ms	$33.8_{-12.5}^{+14.0}$ ms	$2.84_{-1.04}^{+1.59}$ ms	$^{289}115$	10.45(9)	0.22 s	$0.61_{-0.26}^{+0.47}$ s	$0.045_{-0.016}^{+0.033}$ s
$^{285}113$	9.88(8)	5.5 s	$5.49_{-2.28}^{+3.94}$ s	$0.37_{-0.09}^{+0.26}$ s	$^{285}113$	9.61(11)	5.5 s	$35.02_{-18.73}^{+41.28}$ s	$2.08_{-1.11}^{+2.36}$ s
$^{294}117$	10.96(10)	78 ms	$25.4_{-11.3}^{+20.6}$ ms	$36_{-16}^{+32}$ ms	$^{290}115$	10.09(40)	0.016 s	$1.79_{-1.66}^{+26.15}$ s	$1.68_{-1.55}^{+22.99}$ s
$^{286}113$	9.77(10)	19.6 s	$4.30_{-2.12}^{+4.27}$ ms	$2.9_{-1.4}^{+2.8}$ ms	$^{282}\text{Rg}$	9.13(10)	0.51 s	$114.9_{-60.1}^{+131.0}$ s	$45.6_{-24.9}^{+53.6}$ s
$^{278}\text{Mt}$	9.69(19)	7.6 s	$0.60_{-0.43}^{+1.56}$ ms	$0.21_{-0.14}^{+0.55}$ ms	$^{274}\text{Bh}$	8.93(10)	0.9 min	$0.55_{-0.28}^{+0.62}$ min	$7.8_{-4.1}^{+8.4}$ s
$^{289}114$	10.01(3)	$0.97_{-0.32}^{+0.97}$ s	$3.84_{-0.69}^{+0.86}$ s	$0.35_{-0.06}^{+0.08}$ s	$^{288}114$	10.09(3)	$0.47_{-0.12}^{+0.24}$ s	$0.72_{-0.13}^{+0.15}$ s	$0.09_{-0.01}^{+0.02}$ s
$^{285}\text{Cn}$	9.34(3)	$30_{-10}^{+30}$ s	$82.2_{-16.0}^{+20.0}$ s	$6.40_{-1.20}^{+1.48}$ s	$^{281}\text{Ds}$	8.86(3)	$140_{-90}^{+510}$ s	$601_{-124}^{+158}$ s	$43.61_{-8.6}^{+10.67}$ s

between theoretical calculations and experimental data are about two orders of magnitude. It seems that the theoretical results of the GLDM and DDM3Y are consistent with each other, and the experimental data with higher statistics are needed to determine the half-lives of SHN with better accuracy. For  $^{274}\text{Bh}$ , our result coincides with the experimental value, but the calculation of DDM3Y is about 10 times larger than the experimental observation.

When we come to the isotopes  $^{289}114$  and  $^{288}114$  and their decay products in the lower part of Table I, one find that the experimental data are well reproduced by the GLDM. The calculations of the DDM3Y are 3–8 times smaller than experimental results.

It is very valuable to test the experimental data of the most recently produced neutron-deficient isotope  $^{285}114$  using  $^{48}\text{Ca}$  irradiation of  $^{244}\text{Pu}$  targets at Berkeley [4]. As we mentioned in the beginning of this paper, the rms deviation of  $\alpha$ -decay energies for 46 SHN from the MMM is 0.263 MeV. But the present experimental data are not included in the previous fits, which should be a challenge for MMM to reproduce these  $\alpha$ -decay energies. So the  $\alpha$ -decay energies from MMM are also show in Table II. We can see that the experimental  $\alpha$ -decay half-lives are reproduced perfectly by the GLDM when the experimental  $\alpha$ -decay energies are adopted, which implies that the present experimental data are consistent with themselves and that the GLDM can predict the  $\alpha$ -decay half-lives correctly. One may also find that the agreement between the calculated  $\alpha$ -decay energies by the MMM and experimental energies are excellent. The largest deviation is 0.313 MeV for the nucleus  $^{269}\text{Sg}$ . But when we check the  $\alpha$ -decay half-time, the discrepancy is more than one order of magnitude, indicating the theoretical  $\alpha$ -decay energies should be further improved for SHN to strength the theoretical prediction power. For the other nuclei of the  $^{285}114$  decay chain, the theoretical half-times

calculated using the  $\alpha$ -decay energies of MMM are consistent with the experimental results.

Now let us come to the most interesting  $\alpha$ -decay half-life calculations for the Hs and  $Z = 114$  isotopes. The results calculated by taking the experimental  $\alpha$ -decay energies and theoretical MMM energies are shown by small triangles and circles in Fig. 4, respectively. The experimental  $\alpha$ -decay half-lives are also presented by black dots for comparison. It is evident that the neutron magic number appears at  $N = 162$  for the half-times of the Hs isotopes. For  $Z = 114$  isotopes, the maximum values of  $\alpha$ -decay half-lives stand at the magic neutron number  $N = 184$ . If we check the results in detail, one can find that the calculated  $\alpha$ -decay half-lives from experimental  $Q_\alpha$  coincide with the experimental half-lives almost perfectly, implying that, as long as we have the right  $Q_\alpha$ , the presently used method can give precise results for  $\alpha$ -decay half-lives. The calculated  $\alpha$ -decay half-lives with  $Q_\alpha$  from MMM are reasonably consistent with the experimental data, which tells us that the present method can be used to predict the  $\alpha$ -decay half-lives. The  $\alpha$ -decay half-life of the deformed double magic nucleus  $^{270}\text{Hs}$  calculated by a phenomenological formula is 22 s [3] vs 23.33 s by our calculations using the MMM  $Q_\alpha$  and 15.14 s by using the experimental  $Q_\alpha$  (9.02 MeV [3]). For the spherical double magic nucleus  $^{298}114$ , the  $\alpha$ -decay half-life is 1537588 s (about 18 days) with  $Q_\alpha$  of MMM. It would not exist on earth at all if it was not constantly being produced. Since  $^{298}114$  was predicted as the double magic nucleus in the 1960s, many theoretical calculations have been done and the predicted  $\alpha$ -decay half-lives are between several seconds to many years. Recent typical calculations for the  $\alpha$ -decay half-life of  $^{298}114$  is about 4266 seconds in Ref. [15] with the well-known Viola-Seaborg-Sobiczewski formula using FRDM  $\alpha$ -decay energy. For the updated DDM3Y effective interaction, the

TABLE II. Experimental  $\alpha$ -decay energies (in MeV) and half-lives, theoretical  $\alpha$ -decay energies (in MeV) from the MMM and half-lives by the GLDM of the observed  $^{288,289}114$  [4] decay chains.

Nucleus	$Q_\alpha^{\text{expt}}$	$Q_\alpha^{\text{MMM}}$	$T_\alpha^{\text{expt}}$	$T_\alpha^{\text{GLDM}}$	$T_\alpha^{\text{GLDM}^*}$	Nuclei	$Q_\alpha^{\text{expt}}$	$T_\alpha^{\text{MMM}}$	$T_\alpha^{\text{expt}}$	$T_\alpha^{\text{GLDM}}$	$T_\alpha^{\text{GLDM}^*}$
$^{285}114$	10.54	10.323	0.181 s	0.215 s	0.847 s	$^{281}\text{Cn}$	10.46	10.494	0.140 s	0.082 s	0.066
$^{277}\text{Ds}$	10.72	10.639	8.21 ms	4.05 ms	6.51 ms	$^{273}\text{Hs}$	9.73	9.614	346 ms	389 ms	839 ms
$^{269}\text{Sg}$	8.69	8.377	185 s	128 s	1506 s						



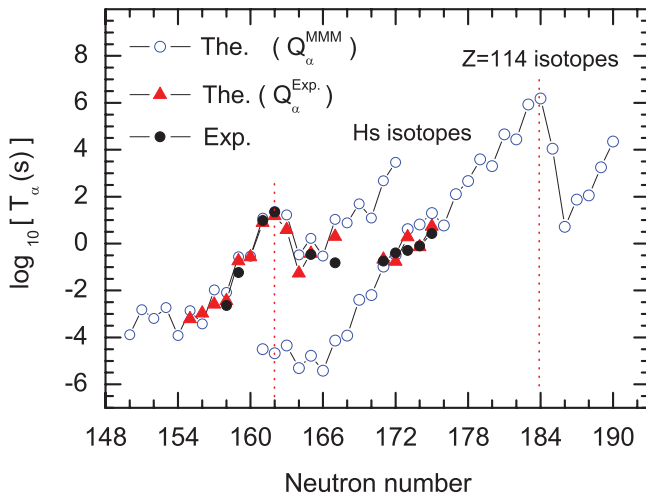


FIG. 4. (Color online) Comparison between experimental  $\alpha$ -decay half-lives and theoretical results.

calculated result is from  $10^3$  to  $10^{13}$  seconds [31] depending on the models for  $\alpha$ -decay energy. We also noted the microscopic Dirac-Brueckner-Hertree-Fock (DBHF) is also adopted to calculate the  $\alpha$ -decay properties and the predicted half-life for  $^{298}114$  is about  $10^4$  s [32], which is smaller than the half-lives of the neighboring neutron-deficient nuclei  $^{292-296}114$ . It seems that different models predicted different  $\alpha$ -decay half-lives. The important question to be considered is how much one can rely on the present predictions. In our calculation, the  $\alpha$ -decay energy can coincide with the known experiment SHN data with an rms of 0.263 MeV. The WKB penetrability with the potential constructed by the GLDM can give the nearly precise  $\alpha$ -decay half-life as long as the right  $\alpha$ -decay energy is adopted. We hope the present calculations will give a relatively trustworthy result for the prediction of  $\alpha$ -decay half-life, helping to synthesize the key nuclide standing at the center of the stability island of the SHN.

#### IV. CONCLUSION AND OUTLOOK

Concluding, a fundamental prediction of modern nuclear theory is the existence of an “island of stability” among the

largely unstable superheavy elements. Different models have predicted different magic numbers and, up to now, this island of stability has not yet been localized experimentally. The central goal of the present work is to find some decisive evidence for localizing this island. With this in mind, we investigate the position of the “island of stability” in a way which is closely connected with the experimental data. The latest experimental average binding energies are compared with the recent calculations by the MMM for the heavy nuclei from Md to  $Z = 118$  elements, and the agreement with the available data is excellent. Both data show that the  $^{270}\text{Hs}$  is a double submagic nucleus after  $^{208}\text{Pb}$ . The features of  $\alpha$ -decay energies and one-proton-separation energies of the MMM reveal that the next double magic nucleus after  $^{270}\text{Hs}$  should be the  $^{298}114$  nucleus. The potential energy surfaces are calculated within the CRMF theory and the results confirm that  $^{270}\text{Hs}$  is a deformed double magic nucleus and the  $^{298}114$  is a spherical double magic nucleus. The  $\alpha$ -decay half-lives are predicted within a generalized liquid drop model and the WKB method and the  $Q_\alpha$  of the MMM for Hs and  $Z = 114$  isotopes, respectively. After finishing the calculation of this work, we noted the report that the heavy-particle radioactivity can be emitted with  $Z_e > 28$  from parents with  $Z > 110$ , implying the cluster radioactivity will be a competitive decay channel with  $\alpha$ -decay and spontaneous fission for superheavy nuclei [33]. It is interesting to estimate the competition between  $\alpha$ -decay and heavy-particle radioactivity in the framework of the present theoretical model, and this work is in progress.

#### ACKNOWLEDGMENTS

We thank Professor D. N. Basu, Zhong-Yu Ma, and Bao-Qiu Chen for sending us their calculated  $\alpha$ -decay half-lives with the DDM3Y interaction and DBHF approach, respectively. H. F. Zhang is grateful to Professor U. Lombardo and W. Zuo for the valuable help. The work was supported by the National Natural Science Foundation of China (Grants No. 10875152, No. 10775061, No. 10825522, and No. 10975064, No. 11105035, and No. 11175074), the CAS Knowledge Innovation Project No. KJCX2-EW-N01.

- [1] S. Hofmann and G. Münzenberg, *Rev. Mod. Phys.* **72**, 733 (2000).
- [2] P. Armbruster, *Eur. Phys. J. A* **7**, 23 (2000).
- [3] J. Dvorak, W. Bröchle, M. Chelnokov *et al.*, *Phys. Rev. Lett.* **97**, 242501 (2006).
- [4] P. A. Ellison, K. E. Gregorich, J. S. Berryman *et al.*, *Phys. Rev. Lett.* **105**, 182701 (2010).
- [5] Yu. Ts. Oganessian, F. Sh. Abdullin, P. D. Bailey *et al.*, *Phys. Rev. Lett.* **104**, 142502 (2010).
- [6] Yu. Ts. Oganessian *et al.*, *Phys. Rev. Lett.* **83**, 3154 (1999); *Eur. Phys. J. A* **5**, 63 (1999).
- [7] Yu. Ts. Oganessian, A. V. Yeremin, A. G. Popeko *et al.*, *Nature (London)* **400**, 242 (1999).
- [8] A. Sobiczewski, F. A. Gareev, and B. N. Kalinkin, *Phys. Lett.* **22**, 500 (1966).
- [9] W. D. Myers and W. J. Swiatecki, *Nucl. Phys.* **81**, 1 (1966).
- [10] S. G. Nilsson *et al.*, *Nucl. Phys. A* **131**, 1 (1969).
- [11] M. Leino and F. P. Hessberger, *Annu. Rev. Nucl. Part. Sci.* **54**, 175 (2004).
- [12] R. D. Herzberg, *J. Phys. G* **30**, R123 (2004).
- [13] R. D. Herzberg, P. T. Greenlees, P. A. Butler *et al.*, *Nature (London)* **442**, 896 (2006).
- [14] Ning Wang, Min Liu, and Xizhen Wu, *Phys. Rev. C* **81**, 044322 (2010); Ning Wang, Zuoying Liang, Min Liu, and Xizhen Wu, *ibid.* **82**, 044304 (2010).
- [15] P. Möller, *At. Data Nucl. Data Tables* **66**, 131 (1997).
- [16] G. Audi, A. H. Wapstra, and C. Thibault, *Nucl. Phys. A* **729**, 337 (2003).
- [17] G. Audi and M. Wang (private communication).
- [18] P. G. Reinhard, *Rep. Prog. Phys.* **52**, 439 (1989).

- [19] B. D. Serot, *Rep. Prog. Phys.* **55**, 1855 (1992).
- [20] P. Ring, *Prog. Part. Nucl. Phys.* **37**, 193 (1996).
- [21] W. Zhang, J. Meng, S. Q. Zhang, L. S. Geng, and H. Toki, *Nucl. Phys. A* **753**, 106 (2005).
- [22] Hongfei Zhang, Junqing Li, Wei Zuo, Zhongyu Ma, Baoqiu Chen, and Soojae Im, *Phys. Rev. C* **71**, 054312 (2005).
- [23] G. A. Lalazissis, J. Konig, and P. Ring, *Phys. Rev. C* **55**, 540 (1997).
- [24] G. Royer and B. Remaud, *Nucl. Phys. A* **444**, 477 (1985).
- [25] H. F. Zhang and G. Royer, *Phys. Rev. C* **76**, 047304 (2007); H. F. Zhang, W. Zuo, J. Q. Li, and G. Royer, *ibid.* **74**, 017304 (2006).
- [26] H. F. Zhang, G. Royer, and J. Q. Li, *Phys. Rev. C* **84**, 027303 (2011).
- [27] H. F. Zhang and G. Royer, *Phys. Rev. C* **77**, 054318 (2008).
- [28] H. F. Zhang, G. Royer, Y. J. Wang, J. M. Dong, W. Zuo, and J. Q. Li, *Phys. Rev. C* **80**, 057301 (2009).
- [29] Ch. E. Düllmann, M. Schädel, A. Yakushev *et al.*, *Phys. Rev. Lett.* **104**, 252701 (2010).
- [30] Partha RoyChowdhury, G. Gangopadhyay, and Abhijit Bhattacharyya, *Phys. Rev. C* **83**, 027601 (2011).
- [31] P. R. Chowdhury, C. Samanta, and D. N. Basu, *Phys. Rev. C* **77**, 044603 (2008).
- [32] Di-Da Zhang, Zhong-Yu Ma, Bao-Qiu Chen, and Shui-Fa Shen, *Phys. Rev. C* **81**, 044319 (2010).
- [33] D. N. Poenaru, R. A. Gherghescu, and W. Greiner, *Phys. Rev. Lett.* **107**, 062503 (2011).