Mass and isospin dependence of symmetry energy coefficients of finite nuclei

Ning Wang[1] and Min Liu[1,2,*]
[1]Department of Physics, Guangxi Normal University, Guilin 541004, People's Republic of China
[2]College of Nuclear Science and Technology, Beijing Normal University, Beijing 100875, People's Republic of China
(Received 25 March 2010; revised manuscript received 2 May 2010; published 7 June 2010)

The mass and isospin dependence of symmetry energy coefficients $a_{\rm sym}$ of finite nuclei are investigated with the measured nuclear masses incorporating the liquid drop mass formula. The enhanced $a_{\rm sym}$ for nearly symmetric nuclei are observed. To describe the mass and isospin dependence of $a_{\rm sym}$, a modified formula based on the conventional surface-symmetry term is proposed and the corresponding rms deviation of nuclear masses is checked.

DOI: 10.1103/PhysRevC.81.067302 PACS number(s): 21.65.Ef, 21.10.Dr, 21.30.Fe, 21.60.Ev

The study of nuclear symmetry energy has attracted great attention in recent years both theoretically and experimentally [1–5]. The mass dependence of symmetry energy coefficients are clearly observed and the obtained symmetry energy coefficients of finite nuclei are considerably smaller than the symmetry energy coefficient of nuclear matter at saturation density. The symmetry energy coefficient of finite nuclei is usually extracted by directly fitting the measured nuclear masses with different versions of the liquid drop mass formula. Some different forms for describing the mass dependence of symmetry energy coefficients of finite nuclei, which divide the symmetry energy of a nucleus into the volume and surface contributions, were proposed in Refs. [3–5]. The volume symmetry energy of the nucleus corresponds to that of nuclear matter at saturation density. In this work, we investigate the symmetry energy coefficients of nuclei, especially the isospin dependence of the symmetry energy coefficient, based on the more than 2000 precisely measured nuclear masses [6].

We start with the well-known liquid drop formula. The liquid drop energy of a nucleus is described by a Bethe-Weizsäcker mass formula [7]

$$E_{\rm LD}(A,Z) = a_v A + a_s A^{2/3} + E_{\rm Coul} + a_{\rm sym} I^2 A, \tag{1}$$

neglecting the pairing term, where $I = (N-Z)/A$ denotes the isospin asymmetry. The symmetry energy coefficient $a_{\rm sym}$ is conventionally expressed as a function of mass number A [3]. The Coulomb energy is written as

$$E_{\rm Coul} = a_c \frac{Z(Z-1)}{A^{1/3}}(1 - 0.76 Z^{-2/3}), \tag{2}$$

with the coefficient $a_c = 0.71$ MeV. Inserting Eq. (2) into Eq. (1) and using the relation $Z = \frac{A}{2}(1-I)$, the liquid drop energy per particle $\varepsilon_{\rm LD} = E_{\rm LD}/A$ can be expressed as a function of mass number A and isospin asymmetry I. Performing a partial derivative of $\varepsilon_{\rm LD}(A,I)$ with respect to the isospin asymmetry I, the symmetry energy coefficient can be expressed as

$$a_{\rm sym} = a_{\rm sym}^{(0)} - \frac{I}{2}\frac{\partial a_{\rm sym}}{\partial I} = a_{\rm sym}^{(0)} + a_{\rm sym}^{(1)} + \cdots, \tag{3}$$

with

$$a_{\rm sym}^{(0)} = \left(\frac{\partial \varepsilon_{\rm LD}(A,I)}{\partial I} - \frac{1}{A}\frac{\partial E_{\rm Coul}(A,I)}{\partial I} \right) \Bigg/ (2I). \tag{4}$$

Omitting the microscopic shell and pairing corrections and the corrections from nuclear deformation, the values of $\frac{\partial \varepsilon_{\rm LD}(A,I)}{\partial I}$ can be obtained from the measured energy per particle $\varepsilon_{\rm exp}$ [6],

$$\frac{\partial \varepsilon_{\rm LD}(A,I)}{\partial I} \approx \frac{\varepsilon_{\rm exp}(A,I_2) - \varepsilon_{\rm exp}(A,I_1)}{I_2 - I_1}. \tag{5}$$

Where I_1 and I_2 denote the isospin asymmetry of nuclei $(A, Z-\Delta Z)$ and $(A, Z+\Delta Z)$, respectively. We take $\Delta Z = 1$ in this work. Through iterations to $a_{\rm sym}$ in Eq. (3),

$$a_{\rm sym} = a_{\rm sym}^{(0)} - \frac{I}{2}\frac{\partial}{\partial I}\left[a_{\rm sym}^{(0)} - \frac{I}{2}\frac{\partial}{\partial I} \times \left(a_{\rm sym}^{(0)} - \frac{I}{2}\frac{\partial}{\partial I}\left[a_{\rm sym}^{(0)} - \cdots \right] \right) \right], \tag{6}$$

one can obtain the expression of $a_{\rm sym}^{(1)}$, which includes all terms with $\frac{\partial}{\partial I} a_{\rm sym}^{(0)}$ in Eq. (6),

$$a_{\rm sym}^{(1)} = \sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^n I \frac{\partial a_{\rm sym}^{(0)}}{\partial I} = -\frac{I}{3}\frac{\partial a_{\rm sym}^{(0)}}{\partial I}. \tag{7}$$

In Fig. 1(a), we show the extracted symmetry energy coefficients of nuclei as a function of nuclear mass number. The crosses and the short dashes denote the extracted $a_{\rm sym}^{(0)}$ and $a_{\rm sym}^{(1)}$ terms of the symmetry energy coefficients from the measured nuclear masses, respectively. One can see that the contribution of the $a_{\rm sym}^{(1)}$ term is much smaller than that of $a_{\rm sym}^{(0)}$ term for most nuclei since I is a small quantity. The relatively large fluctuations in $a_{\rm sym}^{(1)}$ for heavy nuclei are mainly caused by the shell effects. The shades denote the results of Danielewicz *et al.* [3]

$$a_{\rm sym} = c_{\rm sym}[1 + \kappa A^{-1/3}]^{-1}. \tag{8}$$

The extracted $a_{\rm sym}^{(0)}$ term of the symmetry energy coefficients for heavy nuclei are comparable to the results of Danielewicz *et al.* For intermediate and light nuclei, there exist obvious oscillations and fluctuations in the extracted $a_{\rm sym}^{(0)}$, which are probably caused by the shell effects and other nuclear structure effects. In the region $A < 120$, the extracted $a_{\rm sym}^{(0)}$

*Corresponding author: lium_816@hotmail.com

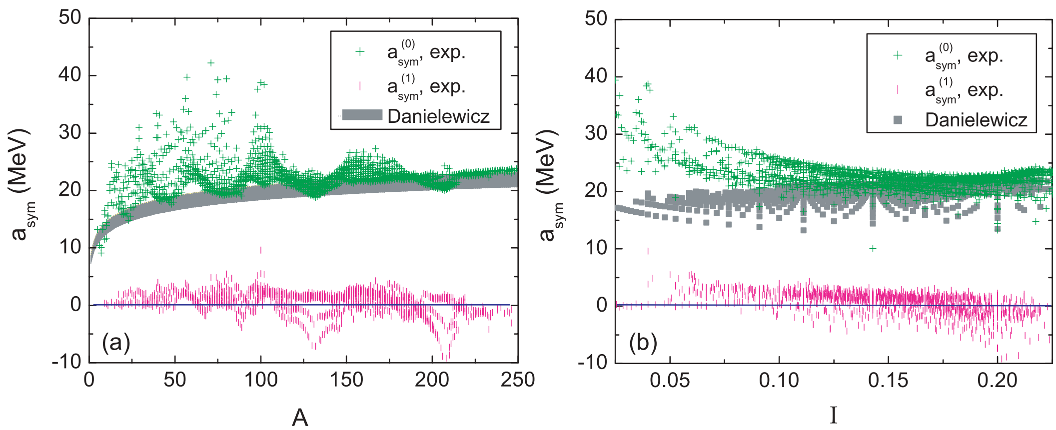

FIG. 1. (Color online) Symmetry-energy coefficients of nuclei as a function of (a) nuclear mass number and (b) of isospin asymmetry. The shades denote the results of Danielewicz *et al.* [3]. The crosses and the short dashes denote the extracted $a_{\rm sym}^{(0)}$ and $a_{\rm sym}^{(1)}$ terms of the symmetry energy coefficients from the measured nuclear masses, respectively.

are generally higher than the results of Danielewicz *et al.* In Fig. 1(b), we show the same data as in Fig. 1(a), but as a function of isospin asymmetry I. One can find that the obtained $a_{\rm sym}^{(0)}$ term of the symmetry energy coefficients somewhat depend on the corresponding isospin asymmetry of nuclei, especially for nearly symmetric nuclei. The $a_{\rm sym}^{(0)}$ term of the symmetry energy coefficient obviously increases with the decrease of asymmetry. The dependence of the symmetry energy coefficient on the asymmetry of the nucleus, especially that $a_{\rm sym}$ increases with increasing proton fraction of the system, is also found in Ref. [1]. The results from Danielewicz *et al.* cannot reproduce the observed trend of isospin dependence well.

To describe the isospin and mass dependence of symmetry energy coefficient of the nucleus, we propose a modified formula,

$$a_{\rm sym} = c_{\rm sym}\left[1 - \frac{\kappa}{A^{1/3}} + \frac{2-|I|}{2+|I|A}\right], \tag{9}$$

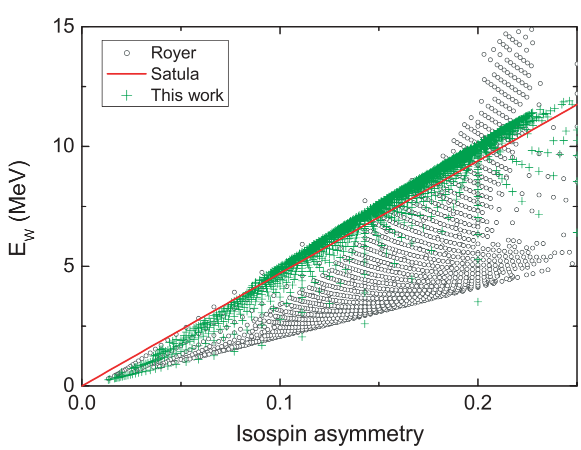

FIG. 2. (Color online) Wigner energies of nuclei calculated with different models. The open circles and the straight line denote the results in Ref. [10] and those of Satula *et al.* [11], respectively. The crosses denote results of this work, with $c_{\rm sym} = 29.3$ MeV determined by fitting the 2149 measured nuclear masses.

based on the conventional surface-symmetry term of the liquid drop model, with a small correction term from isospin asymmetry. The introduced correction term approximately describes the Wigner effect [8–12] of nuclei. The introduced I term in $a_{\rm sym}$ roughly leads to a correction E_W to the binding energy of the nucleus

$$E_W = c_{\rm sym} I^2 A\left[\frac{2-|I|}{2+|I|A}\right] \approx 2c_{\rm sym}|I| - c_{\rm sym}|I|^2 + \cdots, \tag{10}$$

which is known as the Wigner term. In Fig. 2, we show the Wigner energies of nuclei calculated with different models. The open circles and the crosses denote the results in Ref. [10] and those of this work, respectively. The straight line denotes the results of Satula *et al.* [11] (i.e., $E_W \approx 47|I|$). In Ref. [12], Myers and Swiatecki wrote the Wigner term as $E_W = -C_0 \exp[-W|I|/C_0] \approx -C_0 + W|I| + \cdots$, with $C_0 = 10$ MeV, $W = 42$ MeV. The results of this work are comparable to those from Ref. [11] for most nuclei.

With the increasing of mass number A, the $a_{\rm sym}$ in Eq. (9) has a finite value that approaches $c_{\rm sym}$, which corresponds to the symmetry energy coefficient of nuclear matter at saturation density. The results from Eq. (9) are shown in Fig. 3 for comparison, with $c_{\rm sym} = 31$ MeV and $\kappa = 2$, which are obtained by fitting the extracted $a_{\rm sym}^{(0)}$. One can see that the extracted $a_{\rm sym}^{(0)}$ can be reproduced reasonably well. Furthermore, we checked the rms deviations of 2149 masses of nuclei with N and $Z \geqslant 8$ from the measured data defined

TABLE I. Rms σ deviations between 2149 measured data and predictions of Eq. (1) with different $a_{\rm sym}$ forms, and the corresponding optimal parameters of the liquid drop formula.

$a_{\rm sym}$ form	a_v (MeV)	a_s (MeV)	a_c (MeV)	$c_{\rm sym}$ (MeV)	κ	σ (MeV)
Danielewicz	−15.55	18.18	0.71	27.39	1.28	2.71
$c_{\rm sym}[1-\kappa A^{-1/3}]$	−15.57	18.25	0.71	26.09	0.80	2.72
This work	−15.56	18.11	0.71	29.38	1.52	2.55

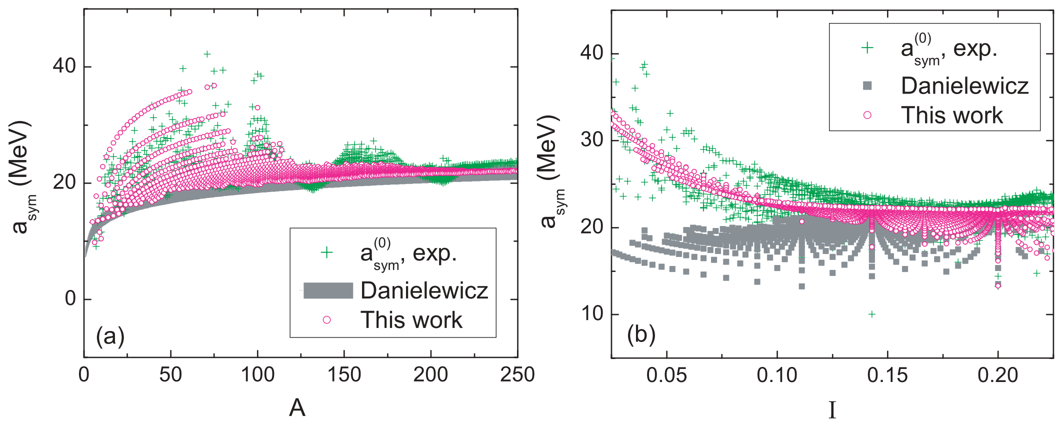

FIG. 3. (Color online) The same as Fig. 1, but with the results of Eq. (9) (open circles) for comparison.

as $\sigma^2 = \frac{1}{m}\sum(M^{(i)}_{\rm exp} - M^{(i)}_{\rm th})^2$ by taking the different forms of the symmetry energy coefficients mentioned previously incorporating the liquid drop mass formula of Eq. (1). The obtained rms deviations and the corresponding parameters of the liquid drop formula are listed in Table I. Adopting the form in Eq. (8), we obtain an rms deviation of 2.71 MeV. With Eq. (9) for the symmetry energy coefficient, the rms deviation is reduced to 2.55 MeV. Compared with the case without the I term being taken into account, the rms deviation is reduced by 6% (see Table I). Incorporating the semi-empirical mass formula in Ref. [13], the rms deviation of the 2149 masses of nuclei can be considerably reduced, falling to 0.516 MeV. Furthermore, when the isospin dependence of the symmetry energy coefficient is taken into account, the obtained optimal $c_{\rm sym}$ changes from 26.09 to 29.38 MeV, which is close to the calculated symmetry energy coefficient of nuclear matter at saturation density from the Skyrme energy density functional [13].

In summary, the mass and isospin dependence of symmetry energy coefficients $a_{\rm sym}$ of finite nuclei was investigated with the measured nuclear masses incorporating the liquid drop formula. For heavy nuclei, the extracted $a^{(0)}_{\rm sym}$ term of the symmetry energy coefficients are consistent with the results of Danielewicz *et al.* For light and intermediate nuclei, there exist oscillations and fluctuations in the extracted $a^{(0)}_{\rm sym}$. The isospin dependence of symmetry energy coefficients, especially the enhanced $a_{\rm sym}$ in nearly symmetric nuclei, was observed. To describe the mass and isospin dependence of $a_{\rm sym}$, we propose a modified formula based on the conventional surface-symmetry term, with which the isospin dependence of $a_{\rm sym}$ can be described reasonably well and the rms deviation of nuclear masses from the experimental data can be effectively reduced.

This work was supported by National Natural Science Foundation of China, Grant Nos. 10875031 and 10847004.

[1] S. K. Samaddar, J. N. De, X. Vinas, and M. Centelles, Phys. Rev. C **76**, 041602(R) (2007).
[2] V. M. Kolomietz and A. I. Sanzhur, Phys. Rev. C **81**, 024324 (2010).
[3] P. Danielewicz *et al.*, Nucl. Phys. A **818**, 36 (2009).
[4] M. W. Kirson, Nucl. Phys. A **798**, 29 (2008).
[5] J. Mendoza-Temis, J. G. Hirsch, and A. P. Zuker, arXiv:0912.0882v1.
[6] G. Audi, A. H. Wapstra, and C. Thibault, Nucl. Phys. A **729**, 337 (2003).
[7] H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. **8**, 82 (1936).
[8] P. Möller *et al.*, At. Data Nucl. Data Tables **59**, 185 (1995).
[9] D. Lunney, J. M. Pearson, and C. Thibault, Rev. Mod. Phys. **75**, 1021 (2003).
[10] G. Royer, Roman. Rep. Phys. **59**, 625 (2007).
[11] W. Satula *et al.*, Phys. Lett. B **407**, 103 (1997).
[12] W. D. Myers and W. J. Swiatecki, Nucl. Phys. A **612**, 249 (1997).
[13] N. Wang, M. Liu, and X. Wu, Phys. Rev. C **81**, 044322 (2010).