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Statistical Behaviors of Quantum Spectra in Superheavy Nuclei*

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Abstract *From the point of view of the interplay between order and chaos, the most regular single-particle motion of neutrons has been found in the superheavy system with $Z = 120$ and $N = 184$ based on the Skyrme–Hartree–Fock model and in the system with $Z = 120$ and $N = 172$ based on the relativistic mean-field model. It has been shown that the statistical analysis of spectra can give valuable information about the stability of suprheavy systems. In addition it may yield deep insight into the single-particle motion in the mean field formed by the superheavy system.*

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1 Introduction

To predict the existence of shell-stabilized superheavy nuclei has been a strong motivation for heavy-ion physics. In the recent three decades with the rapid development of experimental facilities more and more new isotopes have been produced and the expected magic proton number $Z = 114$ seems to be in reach.[1,2] Superheavy elements are thus a topic of current interest and it is worth while to look at them using various theoretical approaches.

The macroscopic-microscopic model was developed quite early. It is based on a generalized liquid-drop model that governs the bulk properties and a single-particle potential from which the shell correction is obtained.[3,4] An alternative macroscopic-microscopic approach is the self-consistent mean-field model. The most widely used self-consistent mean-field models in nuclear physics are the Skyrme–Hartree–Fock (SHF)[5] approach and the relativistic mean-field model (RMF).[6,7]

Based on the above mean-field models one can calculate the shell corrections, and predict the location of expected magic superheavy nuclei. The shell correction resulting from the fluctuation of the energy levels very close to Fermi level provides us with information of shell closures. An alternative shell correction is provided by the statistical study of spectra. As is well known, the statistical property of quantum spectra is closely related to the coupling between energy levels, which depends on the symmetry and structure of the mean field. If the system under consideration has a high degree of symmetry resulting in strong degeneracy of energy levels, the nearest neighboring level spacing distribution will obey a Poisson distribution. This means that the system is quite stable against chaos. If the symmetry of system is broken, a coupling between levels (i.e. their mutual repulsion) appears, and the level-spacing distribution becomes a Wigner distribution. This means that the system will be unstable against chaos.

From the point of view of the interplay between order and chaos, the statistical features of spectra for low-lying nuclear energy levels and for energy levels at excitation energy around the neutron separation energy have been well studied both experimentally and theoretically.[8−13] Based on the realistic shell model calculations the spectral statistics of calcium isotopes has been demonstrated.[14] It has also been reported[15] that for a prolate deformation virtually no chaos is discernible while for the oblate case the motion shows strong chaos when the octupole term is turned on.

Recently, the statistical features of spectra for the deformed space explored by the fission have been studied and a new insight into fission and hyperdeformation has been given.[16] The extension of this kind of investigations to superheavy nuclear systems is very valuable. Through analysis of statistical properties of spectra in superheavy nuclei we expect to obtain new insight into the behavior of the single-particle motion in the mean field formed by superheavy nuclear systems and, furthermore, to get important information about the stability of systems against chaos, i.e. the possible existence of shell-stabilized superheavy nuclei.

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2 Method

Aiming for this goal, in this paper we will study the nearest neighbor level-spacing distributions in superheavy systems based on mean-field models. In order to calculate the energy levels of superheavy nuclear systems, the SHF approach and the relativistic mean-field models are employed.[17,18] In the SHF model, nucleons are described as non-relativistic particles moving independently in a common self-consistent field. The energy functional of nucleons is

$$\varepsilon_{\rm SHF} = \varepsilon_{\rm kin} + \varepsilon_{\rm sk} + \varepsilon_{\rm sk,sl} + \varepsilon_c + \varepsilon_{\rm pair} - \varepsilon_{\rm cm}\,, \qquad (1)$$

where $\varepsilon_{\rm kin}$ is the kinetic energy functional, $\varepsilon_{\rm sk}$ is the Skyrme functional, $\varepsilon_{\rm sk,sl}$ is the spin-orbit term, ε_c is the Coulomb energy (including the exchange term), $\varepsilon_{\rm pair}$ is the pairing energy, and $\varepsilon_{\rm cm}$ is the center of mass correction. In the RMF model, nucleons are described as independent Dirac particles moving in local isoscalar-scalar, isoscalar-vector, and isovector-vector mean fields usually associated with σ, ω, and ρ mesons, respectively. The energy functional of nucleons in the RMF model,

$$\varepsilon_{\rm RMF} = \varepsilon_{\rm kin} + \varepsilon_\sigma + \varepsilon_\omega + \varepsilon_\rho + \varepsilon_c + \varepsilon_{\rm pair} - \varepsilon_{\rm cm}\,, \qquad (2)$$

is composed of the kinetic energy of nucleons, the interaction energies of the σ, ω, and ρ fields, and the Coulomb energy. The pairing energy and center of mass correction are treated in the same way as in the SHF model.

The single-particle energies needed in the statistical analysis of spectra are eigenvalues of the one-body Hamiltonion of the nucleons obtained by variation of the energy functional (1) or (2). The SHF and RMF calculations are carried out using the coordinate-space codes of Refs. [19] and [20]. Since the level density of neutrons (protons) changes greatly with the level energy, we introduce an unfolding procedure[21] in order to study local statistical properties of levels, such as the level spacing distribution. This means that for a given stretch of levels (with the same angular momentum and parity), one has to divide them into many sets. In each set, the local mean level spacing $\bar{S}$ is calculated by the energy interval and the number of levels within the set. From this mean spacing, the level spacing distribution for a given set of levels is readily available. The resultant distribution is obtained by an ensemble average.

To quantify the regularity (or chaoticity) of the level spacing distribution in terms of a parameter, we compare it with the Brody distribution

$$P(s,\omega) = \alpha(\omega+1)s^\omega \exp(-\alpha s^{\omega+1})\,, \qquad (3)$$

where

$$\alpha = \Big[\Gamma\Big(\frac{\omega+2}{\omega+1}\Big)\Big]^{\omega+1}\,. \qquad (4)$$

This distribution interpolates between the Poisson distribution ($\omega = 0$) of regular systems and the Wigner distribution ($\omega = 1$) of chaotic ones (GOE). The parameter ω can be used as a simple quantitative measurement of the degree of regularity (or chaoticity), that is, the smaller ω is, the larger the stability of the corresponding system is.

3 Results and Discussions

Since we use the coordinate space code to make the SHF and RMF calculations, it is necessary to check whether the size of the coordinate-space box is sufficient. To do so, we examine the influence of the box size on the energy levels and statistical results of spectra for ^{208}Pb. As is well known, ^{208}Pb is a very stable nucleus because of the doubly magic structure. Therefore, the typical Poisson distribution of the level spacings is expected in this case, which provides a good check for the minimum box size needed in the model calculations.

In Fig. 1 we show the statistical properties of the spectra by histograms for different sizes of coordinate-space box for ^{208}Pb, where the energy levels are produced by the RMF model with force of NL-Z2. For comparison, Poisson and Wigner distributions are also shown by dashed and dotted lines, respectively. Furthermore, to quantify the regularity of level spacing distributions the best fit Brody parameters are also denoted in each of the subfigures. Clearly with increasing the size of the coordinate-space box the level spacing distribution gradually approaches a Poisson distribution and the best-fit Brody parameters approach zero. When the size of the coordinate-space box is larger than 22 fm (Figs. 1(c) and 1(d)), one can obtain a very regular state (stable state) for ^{208}Pb. However, for the case of a smaller box (see Figs. 1(a) and 1(b)) the level-spacing distribution greatly deviates from the Poisson distribution, which is in conflict with the well-known fact that ^{208}Pb is very stable. This means that the results obtained in these cases are not physically correct and cannot be accepted.

We would therefore like to mention that the statistical analysis of spectra is one of the most effective approaches for checking whether the coordinate or configuration in mean-field calculations is large enough or not. Thus, in the following SHF and RMF calculations we always set the size of the coordinate-space box larger than 22 fm.

According to the shell correction study based on SHF model calculations,[17,18,22] the spherical magic neutron number in the superheavy element region is about $N = 184$ and all isotones with $N = 184$ have been predicted to have spherical shapes. The $N = 172$ shell effect is also strong, but it exhibits an evident force dependence. In Refs. [17], [18], and [22], the proton shell corrections were also studied and it was found that the proton shell corrections are generally smaller than those for the neutrons. With this knowledge, we choose the systems with a fixed proton number $Z = 120$ and neutron numbers $N = 150, 158, 164, 172, 182, 184$, and 190, respectively, for this investigation.

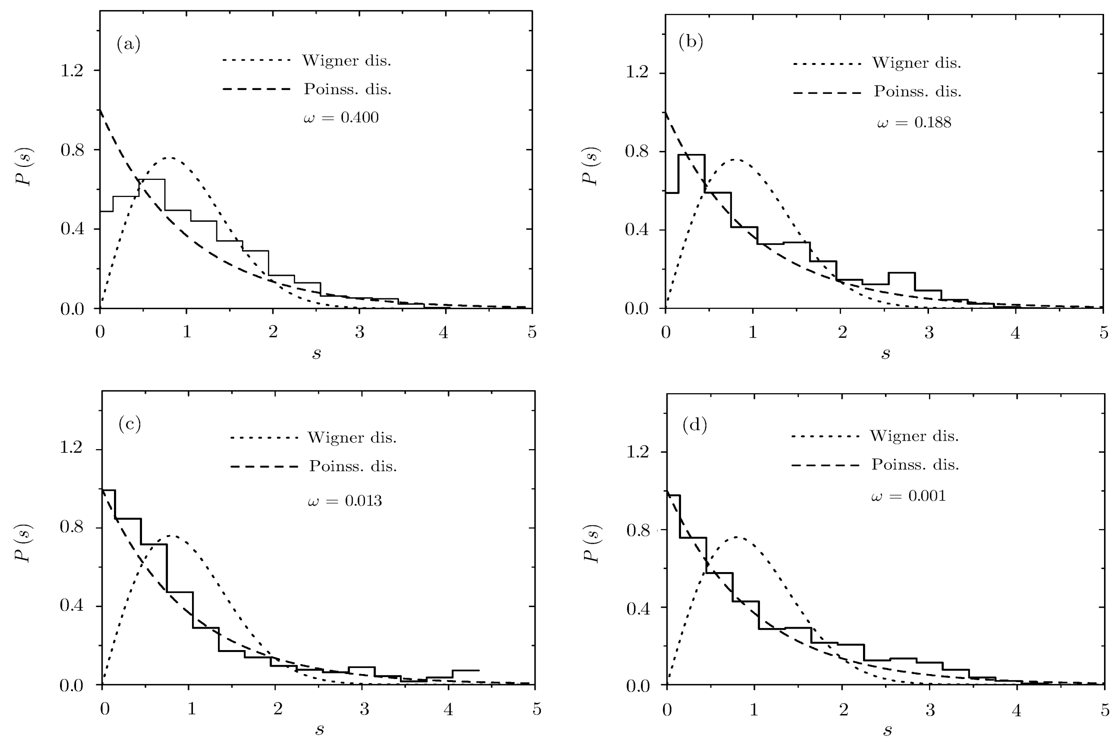

Fig. 1 The nearest-neighbor level-spacing distributions of ^{208}Pb for different sizes of coordinate-space boxes. Subfigures (a), (b), (c), and (d) stand for box sizes of 12, 17, 22, and 30 fm, respectively. The histograms are our numerical results. The dashed lines and dotted lines represent a Poisson distribution and Wigner distribution, respectively. The best-fit Brody parameters are also indicated in each of the subfigures.

As the first step to study the stability of superheavy systems from the statistical aspect of the spectra, we perform calculations with the SHF model with several different versions of the Skyrme force including the force SLY6 and then make systematic statistical analysis of the neutron levels for all the above systems. As an example, we show the nearest-neighbor level-spacing distributions of the neutron spectra for systems with neutron numbers $N = 158, 172, 184$, and 190 in Fig. 2. The histograms represent our numerical results and for comparison we also draw the Poisson and Wigner distributions in dashed and dotted lines.

To quantify the regularity of the level-spacing distributions we notice the Brody parameters in each of the subfigures. It can be seen that with increasing neutron numbers from 158 to 172 and then to 184 the level spacing distributions of the neutrons gradually approach a typical Poisson distribution (see Figs. 2(a), 2(b), and 2(c)). For the case with even more neutrons, such as $N = 190$, however the distribution deviates from the Poisson distribution (see Fig. 2(d)). Therefore the general tendency is that the regularity shown by the level spacing distributions keeps increasing up to $N = 184$, but after $N = 184$, this trend stops and the regularity decreases. From this study we may draw a conclusion that for the system of $Z = 120$ and $N = 184$ the most regular single-particle motion of neutrons is expected in the mean field formed by superheavy systems based on the SHF model. This means that the corresponding system is more stable than the other systems with the same proton number but different neutron numbers.

The RMF model was also used to investigate the possible existence of superheavy systems. In this study there exist many parameter sets which differ in details. For the purpose of the present study, we chose one of the most successful parameter sets for heavy nuclear systems, NL-Z2, to perform RMF calculations. In Refs. [17] and [18] it was predicted that the minimum of the neutron shell correction is located at $N = 172$ and the proton shell correction at $Z = 120$ is strongly correlated to neutron number $N = 172$ by looking at the proton shell corrections along the chain of $N = 172$ and 184 isotones. Based on this study we choose the same systems as with SHF for investigation.

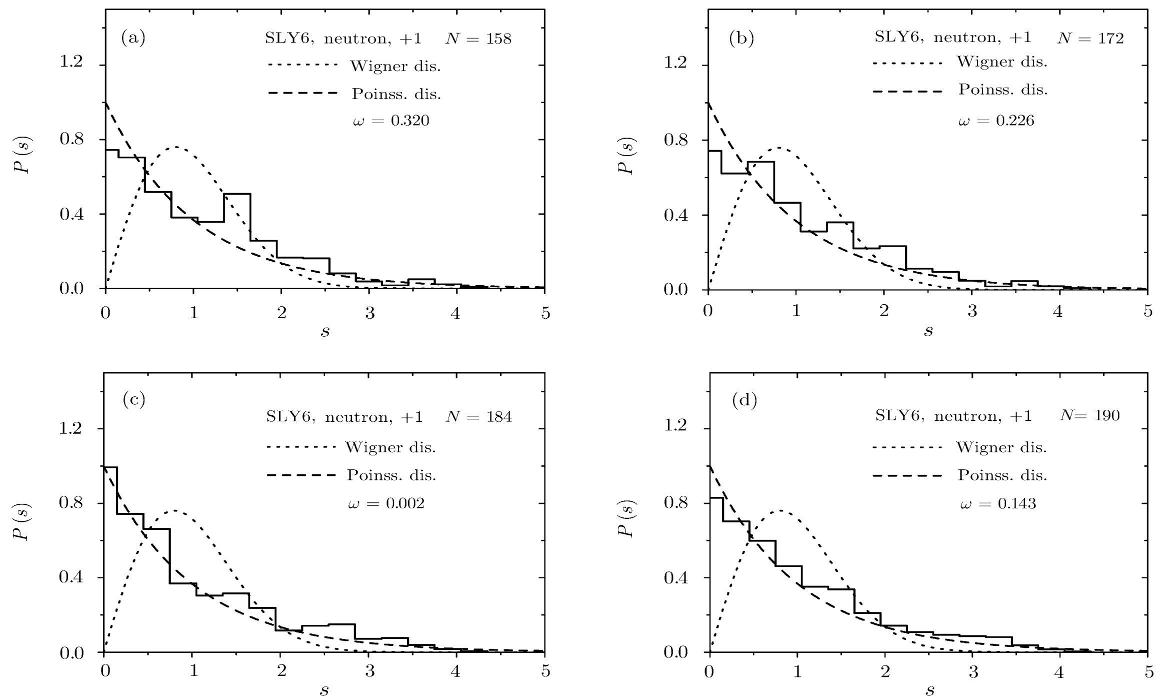

Fig. 2 The nearest-neighbor level-spacing distributions for systems with proton numbers $Z = 120$ and neutron numbers (a) $N = 158$, (b) $N = 172$, (c) $N = 184$, and (d) $N = 190$. The histograms are our numerical results based on the SHF model. The dashed lines and dotted lines represent a Poisson distribution and Wigner distribution, respectively. The best-fit Brody parameters are also indicated in each of the subfigures.

We carried out the statistical analysis of the quantum spectra for those systems systematically. As an example, the level-spacing distributions of neutrons for the systems with $Z = 120$ and $N = 158$, 164, 172, and 184 are displayed by histograms in Fig. 3. The Poisson and Wigner distributions are also shown in each of the subfigures for comparison, and to quantify the regularity the Brody parameters are indicated in each subfigure. From this figure we can see that the regularity shown by the level-spacing distributions first increases and then decreases with increasing neutron number. This behavior is similar to that in the SHF case. The strongest regularity of the statistical results (the typical Poisson distribution of level spacings), however, appears for $N = 172$ in the RMF model and not for $N = 184$ as in the SHF model. This model-dependent difference in prediction of the most stable superheavy systems has also been found in the shell correction study. This implies that there is the relation between the statistical analysis of spectra and the shell correction in characterizing the stability of systems.

In order to demonstrate that the statistical analysis of spectra is a valuable approach in characterizing the stability of systems, let us make a comparison between it and the shell correction approach. As is already mentioned, the shell correction reflecting the stability of systems results from the fluctuation of levels very close to Fermi energy, and the statistical property of spectra depends on the local fluctuation of a set of levels, which relates the single particle motion to the stability of the system against chaos. Based on these considerations, we demonstrate the shell correction and Brody parameters for superheavy systems of $Z = 120$ as functions of neutron numbers in Figs. 4(a) and 4(b), respectively. In each subfigure the solid line represents the results based on the SHF model with the force SLY6, and the dashed line is for the results based on the RMF model with the force NL-Z2. Through the comparison between solid and dashed curves in Figs. 4(a) and 4(b), we have found that there is quite a similar behavior of the Brody parameter and shell correction as functions of neutron number in both the SHF and the RMF. Therefore, we may conclude that the statistical analysis of spectra can indeed give very valuable information on predicting and studying suprheavy systems. Although this study is very preliminary, the significance of this kind of study can go far beyond the investigation on the stability of superheavy systems and may also give a deep insight into the single particle motion in the mean field formed by the superheavy system, which is one of the most fundamental problems in superheavy nuclear study. It may be hoped that this will open up a new research field on the transition from order to chaos in superheavy region.

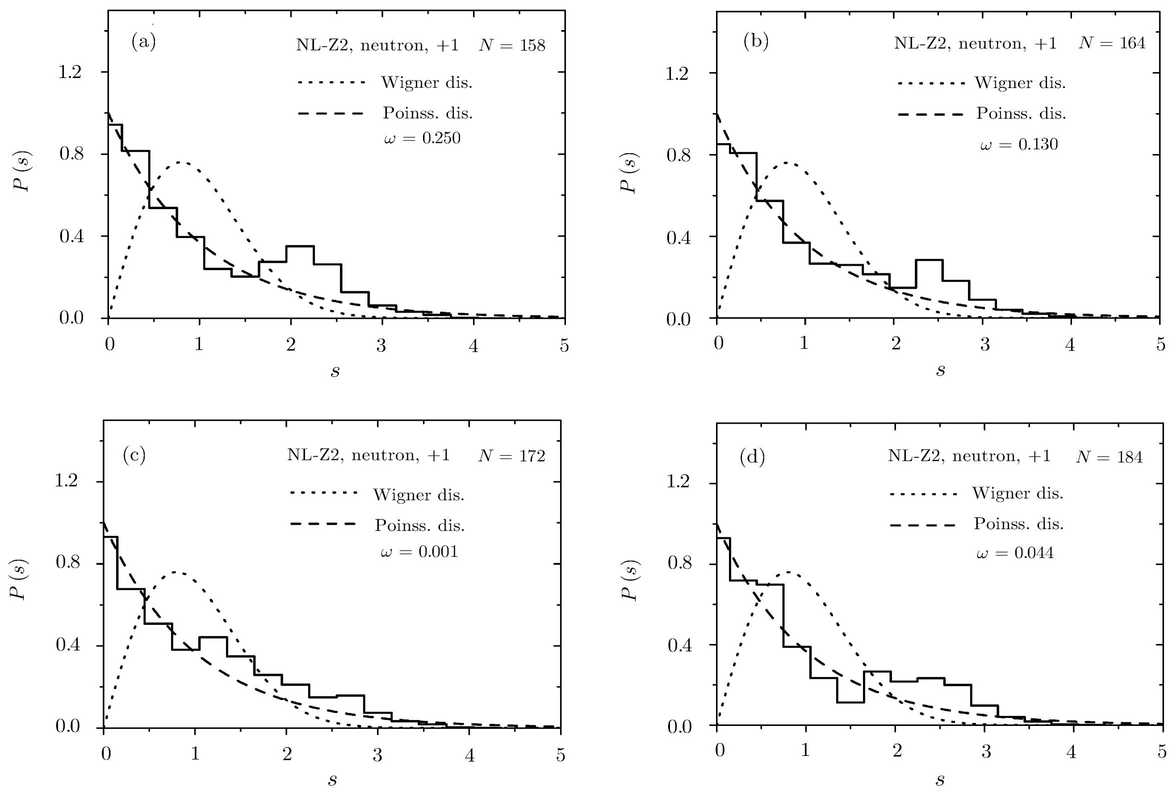

Fig. 3 The same as Fig. 2 for systems with proton numbers $Z = 120$ and neutron numbers (a) $N = 158$, (b) $N = 164$, (c) $N = 172$, and (d) $N = 184$. The histograms are our numerical results based on the RMF model.

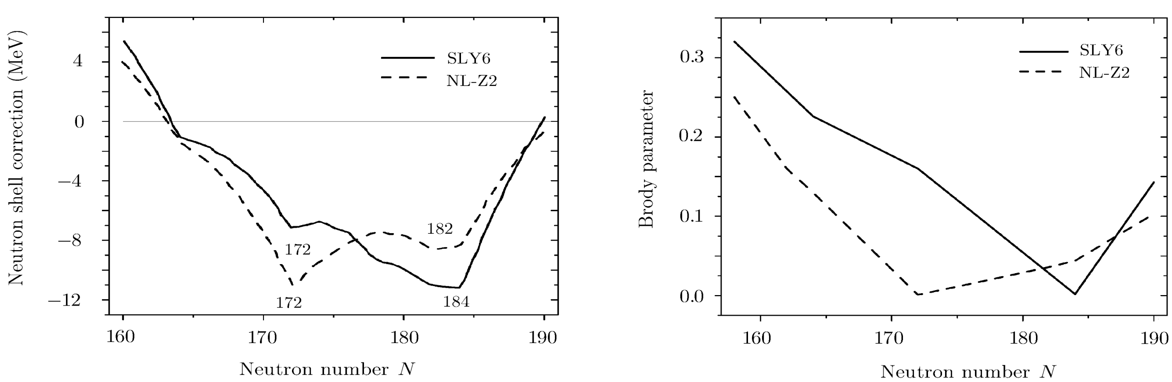

Fig. 4 (a) The neutron shell corrections for $Z = 120$ isotopes calculated in the SHF model with the force SLY6 (solid line) and the RMF model with the force NL-Z2 (dashed line); (b) The best-fit Brody parameter as a function of neutron numbers for $Z = 120$ isotopes. The solid and dashed lines stand for the SHF model with the force SLY6 and the RMF model with the force NL-Z2, respectively.

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