Modified Woods–Saxon Potential for Heavy-Ion Fusion Reaction *

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A modified Woods–Saxon (MWS) potential is proposed for describing nucleus-nucleus interaction based on the Skyrme energy-density functional approach. Fusion barriers for a large number of fusion reactions from light to heavy systems can be described well with this potential. The suitable incident energies for fusion reactions leading to superheavy nuclei are also explored. It seems to us that the MWS potential is useful for selecting the suitable incident energies for fusion reactions for producing super-heavy nuclei.

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Research on nucleus-nucleus interaction potential in heavy-ion fusion reactions has attracted a great deal of attention.[1−7] Some of macroscopic potentials such as Woods–Saxon potential, proximity potential[2] and Bass potential[3] without explicitly considering microscopic properties of nuclei like shell effects and the saturation of nuclear interaction are widely used. Simultaneously, some potentials such as the entrance channel potential,[4] and M3Y potential[5,6] based on microscopic nucleon-nucleon interactions and/or realistic density distributions of nuclei were proposed for describing the nucleus-nucleus interaction microscopically. These models provide more accurate information for heavy-ion reactions at energies near barrier. But, these microscopic potentials are slightly more complicated computationally than those macroscopic potentials, generally.

In Ref. [8], we proposed a method to obtain the entrance channel potential by using the Skyrme energy density functional together with the extended Thomas–Fermi approximation[9](up to second order $\hbar$)(SEDF + ETF2). Fusion reaction cross sections for a large number of systems can be described very well with the fusion barriers obtained with this approach. In this letter, we will propose a modified Woods–Saxon (MWS) potential with the parameters determined from SEDF+ETF2 approach.

Now we briefly introduce the SEDF+ETF2 approach for calculating the interaction potential barrier, a more detailed description can be found in Ref. [8]. The interaction potential $V_b(R)$ between reaction partners can be written as

$$V_b(R) = E_{\rm tot}(R) - E_1 - E_2, \tag{1}$$

where R is the center-to-center distance between reaction partners, $E_{\rm tot}(R)$ is the total energy of the interaction system, E_1 and E_2 are the energies of the non-interacting projectile and target, respectively. The interaction potential $V_b(R)$ is called entrance-channel potential[4] or fusion potential. Here $E_{\rm tot}(R)$, E_1, E_2 are determined by the Skyrme energy-density functional,[9−11]

$$E_{\rm tot}(R) = \int \mathcal{H}[\rho_{1p}(\boldsymbol{r}) + \rho_{2p}(\boldsymbol{r}-\boldsymbol{R}), \rho_{1n}(\boldsymbol{r}) + \rho_{2n}(\boldsymbol{r}-\boldsymbol{R})]\, d\boldsymbol{r}, \tag{2}$$

$$E_1 = \int \mathcal{H}[\rho_{1p}(\boldsymbol{r}), \rho_{1n}(\boldsymbol{r})] d\boldsymbol{r}, \tag{3}$$

$$E_2 = \int \mathcal{H}[\rho_{2p}(\boldsymbol{r}), \rho_{2n}(\boldsymbol{r})] d\boldsymbol{r}, \tag{4}$$

where ρ_{1p}, ρ_{2p}, ρ_{1n} and ρ_{2n} are the frozen proton and neutron densities of the projectile and target, and the expression of the Skyrme energy-density functional $\mathcal{H}$ can be found in Refs. [4,8]. Once the proton and neutron density distributions of the projectile and target are determined, the interaction potential $V_b(R)$ can be calculated from Eqs. (1)–(4).

By density-variational approach and minimizing the total energy of a single nucleus given by the integral of $\mathcal{H}$, the neutron and proton densities of this nucleus can be obtained.[8] The Skyrme force SkM*[11] is adopted in this work. For a certain reaction system, the entrance-channel potential is calculated in the range from R = 7 fm to 15 fm with a step size $\Delta R = 0.25$ fm.

Now let us discuss how to obtain the parameters of the MWS potential from the entrance-channel potential. The nucleus-nucleus potential reads

$$V(R) = V_N(R) + V_C(R), \tag{5}$$

where V_N and V_C are the nuclear interaction and the Coulomb interaction, respectively. We take $V_C(R) = e^2 Z_1 Z_2/R$, and the nuclear interaction V_N to be Woods–Saxon form with five parameters determined by fitting the obtained entrance-channel potentials:

$$V_N(R) = \frac{V_0}{1+\exp[(R-R_0)/a]}, \tag{6}$$

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with[17]

$$V_0 = u_0[1 + \kappa(I_1 + I_2)]\frac{A_1^{1/3}A_2^{1/3}}{A_1^{1/3} + A_2^{1/3}}, \quad (7)$$

and

$$R_0 = r_0(A_1^{1/3} + A_2^{1/3}) + c. \quad (8)$$

$I_1 = (N_1 - Z_1)/A_1$ and $I_2 = (N_2 - Z_2)/A_2$ in Eq. (7) are the isospin asymmetry of projectile and target nuclei, respectively.

By varying five parameters u_0, κ, r_0, c and a of the MWS potential , we minimize the relative deviation between the fusion barrier heights B_0 obtained with the SEDF+ETF2 approach (see Eq. (1)) and the barrier heights of MWS potential $B_{\rm ws}$ obtained with Eq. (5). The relative deviation is defined as

$$s^2 = \frac{1}{m}\sum_{i=1}^{m}\left[(B_0^{(i)} - B_{\rm ws}^{(i)})/B_0^{(i)}\right]^2, \quad (9)$$

The corresponding optimal values of these parameters are obtained when s gets the minimum of the relative deviation. Here m in Eq. (9) is the number of reactions involved. In this work, 66996 reactions with $Z_1Z_2 \leq 3000$ are involved to determine the parameters of the MWS potential. The obtained optimal values of the parameters are listed in Table 1, and the minimal deviation calculated with these parameters is $s^2 = 0.0017$.

Table 1. Parameters of the modified Woods–Saxon potential.

r_0 (fm)	c (fm)	u_0 (MeV)	κ	a (fm)
1.27	−1.37	−44.16	−0.40	0.75

Figure 1 shows the comparison of interaction potentials for reactions ^{16}O+^{92}Zr, ^{28}Si+^{92}Zr, ^{16}O+^{208}Pb and ^{48}Ca+^{208}Pb. The dotted and solid curves denote the results with the SEDF+ETF2 approach and those with the MWS potential, respectively. The results of proximity potential are also presented by crossed curves for comparison. The results with the MWS potential are in good agreement with the potentials calculated with the SEDF+ETF2 approach in the region where two nuclei do not overlap much.

Here we should mention that the entrance channel potential obtained with the SEDF+ETF2 approach is based on frozen-density approximation. Therefore, the MWS potential gives the uncoupled fusion potential which is usually higher than the average fusion barrier extracted experimentally. By using the parameterized barrier distribution proposed in Refs. [8,12], the fusion (capture) excitation function of fusion reaction can be calculated easily based on the MWS potential. Figure 2 shows the fusion excitation functions of reactions ^{16}O+^{92}Zr, ^{28}Si+^{92}Zr, ^{16}O+^{208}Pb and ^{48}Ca+^{208}Pb. The solid and dashed curves denote the results with the MWS potential and those with the SEDF+ETF2 approach, respectively. The results with the two approaches are very close and both can reproduced the experimental data quite well. The relation between the mean barrier height $B_{\rm m}$ of the reaction system obtained from the parameterized barrier distribution and the barrier height of MWS potential is

$$B_{\rm m} \simeq 0.956 B_{\rm ws} \quad (10)$$

when the structure factor of the parameterized barrier distribution[8] $\gamma = 1$ is taken. Here $B_{\rm ws}$ is the barrier height of the MWS potential. From the relation between $B_{\rm m}$ and $B_{\rm ws}$ in Eq. (10) one finds there exists 4.4% barrier reduction due to the coupling effects between relative motion and other degrees of freedom. As an example, for the fusion reaction ^{16}O+^{208}Pb Gontchar *et al.* in ref. [13] found that the inclusion of the coupling to the projectile and target excitation states effectively reduced the uncoupled barrier height by 3.8 MeV in total by using the fusion coupled channel model. The average fusion barrier

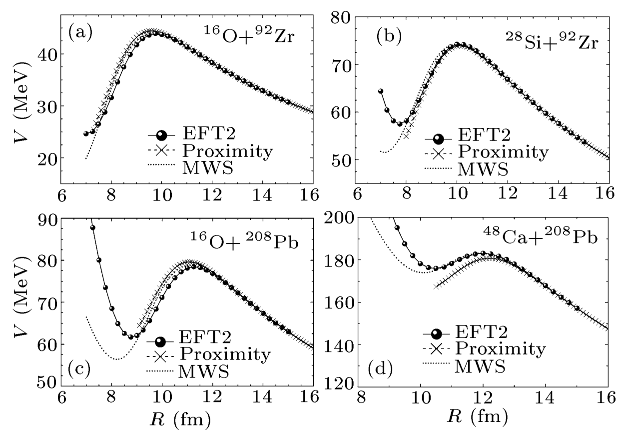

Fig. 1. Interaction potentials for ^{16}O+^{92}Zr, ^{28}Si+^{92}Zr, ^{16}O+^{208}Pb and ^{48}Ca+^{208}Pb. The dotted curves and the solid curves denote the results with SEDF+ETF2 approach and the MWS potential, respectively. The crossed curves denote the results of proximity potential.

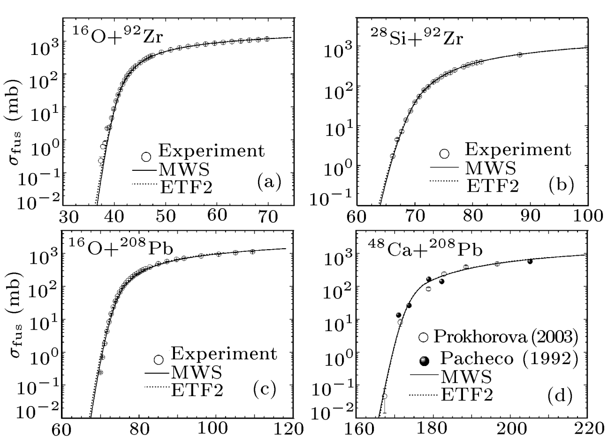

Fig. 2. Fusion (capture) excitation function of reactions ^{16}O+^{92}Zr, ^{28}Si+^{92}Zr, ^{16}O+^{208}Pb and ^{48}Ca+^{208}Pb. The solid and dashed curves denote the results with the MWS potential and with the SEDF+ETF2 approach, respectively. The circles are the experimental data.

height is 74.5 MeV, determined from fitting the above-barrier fusion cross sections with a single barrier model and thus they concluded that the uncoupled barrier height of 78.3 MeV was required. Thus, for this reaction a 4.8% barrier energy shift is found, which is in good agreement with our result of 4.4%. The mean barrier height $B_{\rm m} = 75.3$ MeV is obtained with the MWS potential when the shell effect is neglected and $\gamma = 1$, which is close to the extracted average barrier energy.

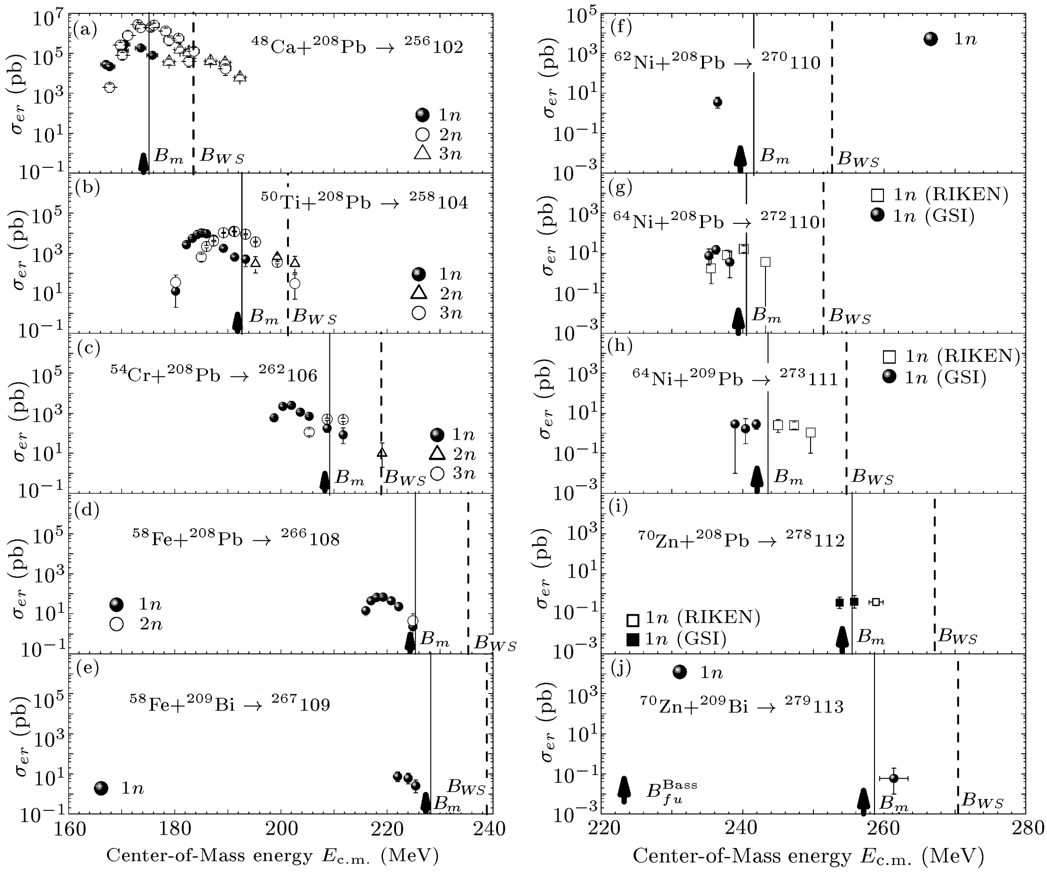

Fig. 3. Experimental data for evaporation cross sections for some "cold fusion" reactions. The solid and the open (in RIKEN) squares denote the measured evaporation-residue cross section for the 1n channel. The open circles and the open triangles denote the data for 2n and 3n channel, respectively. The solid and the dashed lines denote the energies corresponding to the barrier heights $B_{\rm m}$ and $B_{\rm ws}$ obtained with the MWS potential, respectively. The Bass barrier heights $B_{fu}^{\rm Bass}$ denoted by solid arrows are also presented for comparison.

The relative deviation between the mean barrier heights for 47 fusion reactions with $Z_1Z_2 < 1600$ calculated with Eq. (10) by using the MWS potential and those extracted from measured fusion excitation energies[14] are smaller than 2.5% for most of reactions, and smaller than 1% for about 60% reactions. It is indicated that the MWS potential can describe the fusion barriers of heavy-ion reactions well.

For massive fusion reactions, especially for the fusion reactions leading to super-heavy nuclei, it is important to select suitable incident energies. In this work, the suitable incident energies for the "cold fusion" and "hot fusion" in the synthesis of super-heavy nuclei are explored systematically. In Fig. 3, we show the measured evaporation-residue cross sections for 10 cold fusion reactions. The solid and the open (in RIKEN) squares denote the measured evaporation-residue cross sections for the 1n channel, the open circles and the open triangles are the measured evaporation-residue cross sections for the 2n and 3n channels, respectively. The barrier energies obtained with the MWS fusion potential are also presented for comparison. The mean barrier height $B_{\rm m}$ and the fusion barrier height $B_{\rm ws}$ are denoted by the solid line and the dashed line, respectively. The arrows denote the Bass barriers. The data for the evaporation-residue cross sections of 10 cold fusion reactions with Pb or Bi target are taken from Refs. [15–18]. From Fig. 3, one sees that the calculated mean barrier height $B_{\rm m}$ are very close to the Bass barrier height $B_{fu}^{\rm Bass}$,[19] and $B_{\rm m}$ just locate at the peaks of the 2n channel for the reactions with ^{48}Ca, ^{50}Ti, ^{54}Cr and ^{58}Fe. The 1n channel appears to be deeply sub-barrier fusion for the reactions producing nuclei with $Z < 110$. With the increase of projectile mass number, the energies of the 1n channel get close to the calculated mean barrier height $B_{\rm m}$. Here we have not taken into account the effects from deformation, the shell correction of reaction partners as well as the correction term from the angular momentum, which are also important for selecting proper incident energy for producing superheavy nuclei. The study on these effects are in progress.

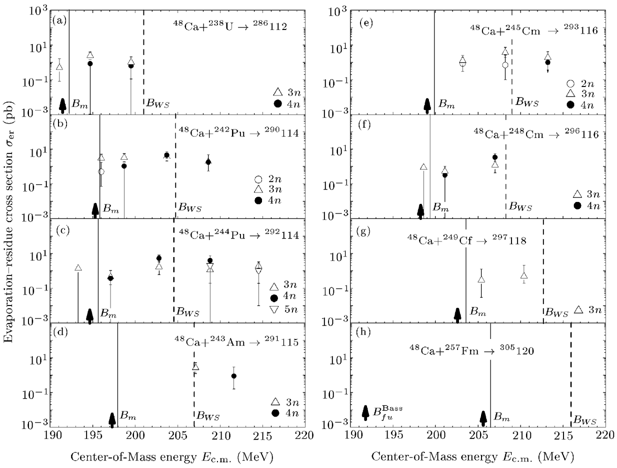

Fig. 4. The same as Fig. 3, but for some hot fusion reactions. The open circles, open triangles, solid circles and down-triangles denote the measured evaporation-residue cross sections for the $2n$, $3n$, $4n$ and $5n$ channels, respectively.

The experimental evaporation-residue cross sections for some hot fusion reactions are shown in Fig. 4. The open circles, open triangles, solid circles and down-triangles denote the measured evaporation-residue cross sections for the $2n, 3n, 4n$ and $5n$ channel measured by Dubna.[20], respectively. The calculated mean barrier height $B_{\rm m}$ are about 1 MeV higher than the Bass barrier $B_{fu}^{\rm Bass}$. From Fig. 4, one finds that the experimental evaporation-residue excitation functions of these fusion reactions are peaked at the energies ranging from $B_{\rm m}$ to $B_{\rm ws}$ for all reactions except the reaction ^{48}Ca+^{243}Am, in which only two data points are available. It seems to us that the MWS potential are useful for selecting the suitable incident energies for producing super-heavy nuclei. It is easy to calculate the $B_{\rm m}$ and $B_{\rm ws}$ for fusion reactions leading to synthesis of super-heavy element Z=120 by using the MWS potential. The calculated values for $B_{\rm m}$ and $B_{\rm ws}$ are 206.5 and 215.8 MeV for ^{48}Ca+^{257}Fm, 223.2 and 233.4 MeV for ^{50}Ti+^{252}Cf, 251.8 and 263.2 MeV for ^{58}Fe+^{244}Pu, and 314.5 and 328.8 MeV for ^{94}Sr+^{208}Pb, respectively.

In summary, we have proposed a MWS potential for describing nucleus-nucleus interaction based on the SEDF+ETF2 approach. Fusion barriers of a large number of fusion reactions from light to heavy systems can be described well with this potential. The fusion excitation functions of reactions ^{16}O+^{92}Zr, ^{28}Si+^{92}Zr, ^{16}O+^{208}Pb and ^{48}Ca+^{208}Pb calculated based on this potential are in good agreement with experimental data. The suitable incident energies for cold and hot fusion reactions leading to superheavy nuclei are also explored. It seems to us that the MWS potential is useful for selecting the suitable incident energies for producing super-heavy nuclei, which are between the mean barrier height $B_{\rm m}$ and the fusion barrier height $B_{\rm ws}$.

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