Behavior of the Lyapunov Exponent and Phase Transition in Nuclei *

WANG Nan(王楠)[1,2], WU Xi-Zhen(吴锡真)[1,4], LI Zhu-Xia(李祝霞)[1,3,4], WANG Ning(王宁)[1,2], ZHUO Yi-Zhong(卓益忠)[1,3,4], SUN Xiu-Quan(孙秀泉)[2]

[1] *China Institute of Atomic Energy, Beijing 102413*

[2] *Department of Physics, Northwest University, Xi'an 710069*

[3] *Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080*

[4] *Nuclear Theory Center of National Laboratory of Heavy Ion Accelerator, Lanzhou 730000*

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Based on the quantum molecular dynamics model, we investigate the dynamical behaviors of the excited nuclear system to simulate the latter stage of heavy ion reactions, which associate with a liquid-gas phase transition. We try to search a microscopic way to describe the phase transition in real nuclei. The Lyapunov exponent is employed and examined for our purpose. We find out that the Lyapunov exponent is one of good microscopic quantities to describe the phase transition in hot nuclei. Coulomb potential and the finite size effect may give a strong influence on the critical temperature. However, the collision term plays a minor role in the process of the liquid–gas phase transition in finite systems.

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Many recent experiments in heavy-ion collisions at energy above 50 MeV/nucleon have shown a very rapid breakup of the hot composite system formed in the reaction into several large fragments wih $Z \geq 3$.[1-3] In Ref. 4, Gilkes *et al.* studied the critical exponents from the multifragmentation of Gold nuclei and found that this exponents were close to the nominal liquid-gas values. In parallel with experiments, there has been an intense theoretical effort in understanding the phenomenon.[5-8] Since the elementary nucleon-nucleon interaction which exhibits a short range repulsion followed by a long range attraction is similar to a Van Der Waals force, it is expected that there should exist an instability region of density and temperature where a liquid-gas phase transition may occur in nuclear matter. Many works based on the thermodynamics and statistics have been done.[9,10] For a real nucleus, as a typical finite system, the investigation on the critical phenomenon is much less than that for nuclear matter, since for those finite systems we firstly need to look for a good microscopic quantity to characterize the liquid-gas phase transition.[11,12] This is a quite general question which does not regard nuclear physics only but any domain of physics dealing with finite systems like metallic clusters, fullerences, etc. Therefore, it is of special interest to explore the origin of the instability region to be linked to a liquid-gas phase transition in real nuclei microscopically. In order to study the critical phenomenon in the real nuclei microscopically, we consider the dynamic evolution of an excited (hot) nuclear system to simulate the latter stage of the heavy ion reactions, that is, the expanding process of the initial excited nuclear systems with the quantum molecular dynamics (QMD) model.[13] Through such investigation we want to learn whether the Lyapunov exponent can characterize the liquid-gas phase transition in real nuclei or not and how the Coulomb, the size of the nuclear system as well as the collision term influence on the critical temperature.

In our calculation the potential considered includes the Skyme, Yukawa, Coulomb, symmetric and momentum dependent terms. The Pauli principle is also taken into account in our numerical simulation. The potential and parameters are taken to be the same with Ref. 14. We create an initially excited nucleus as follows: firstly we prepare a nuclear system in its ground state according to the binding energy and nuclear radius, then the momentum of each nucleon is sampled according to Fermi-Dirac distribution at a certain value of the chemical potential and temperature T. Through varying the temperature, we put the different initial kinetic energy (initial excitation energy) into the nuclear system.

At low initial excitation energy, for instance $T = 5\,\mathrm{MeV}$, the time evolution of the average density of the nucleus Pb oscillates around the normal nuclear matter density ρ_0, as shown by the curve 1 of Fig.1, in which the average density is defined as

$$\langle\rho\rangle = \frac{\int \mathrm{d}\boldsymbol{r}\rho(\boldsymbol{r},t)\rho(\boldsymbol{r},t)}{\int \mathrm{d}\boldsymbol{r}\rho(\boldsymbol{r},t)}, \tag{1}$$

where $\rho(\boldsymbol{r},t)$ is a density. The curve 1 indicates that at this initial excitation energy, the nucleus remains at stable state without expanding and the breakup does not occur. With increasing the initial excitation energy, the completely different situation appears, for example, at $T = 13$, 14, 20 MeV, the average densities of the nucleus decrease rapidly with time and at about the time $t \sim 80-100\,\mathrm{fm/c}$, they fall to the $(0.2\text{-}0.4)\rho_0$, which have been shown in the curves 2, 3 and 4 of Fig. 1. From the experimental study,[15] we know that the breakup density is approximately one-third of the normal nuclear matter density, which means that the critical density of the nucleus is about $(0.2-0.4)\rho_0$. Therefore, the sharp decrease in the density we observed really reflects the process of fragmentation. The low asymptotic constant density shown by curves 2, 3 and 4 may give evident signals that the liquid–gas phase transition has happened.

For a system undergoing such a liquid–gas phase transition, the dynamic evolution is expected to be

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dominated by an exponential growth of the local initial perturbations. This means that two different trajetories of particles (different events), having a small initial relative distance in phase space, will soon diverge exponentially. From this point of view, the Lyapunov Exponent appears as an appropriate microscopic observable to study critical phenomenon for a real nuclear system. In order to calculate the largest Lyapunov exponent, the following metric in phase space

$$d(t) = \sqrt{\sum_{i=1}^{n}[(\boldsymbol{r}_{i01} - \boldsymbol{r}_{i02})^2 + (\boldsymbol{p}_{i01} - \boldsymbol{p}_{i02})^2]} \quad (2)$$

is defined.[15] Here, $\boldsymbol{r}_{i0j}$ and $\boldsymbol{p}_{i0j}$(subscript $j = 1$ or 2 for two different events) are the scaled positions and momenta

$$\boldsymbol{r}_{i0j} = \frac{\boldsymbol{r}_{ij}}{r_{\rm rsm}}, \quad \boldsymbol{p}_{i0j} = \frac{\boldsymbol{p}_{ij}}{p_{\rm ave}}, \quad (3)$$

where $\boldsymbol{r}_{ij}$ and $\boldsymbol{p}_{ij}$ are the position and momentum for i particle, respectively, $r_{\rm rsm}$ and $p_{\rm ave}$ are the root mean square radius and average momentum. The sum runs over all the N particles of the nucleus. Normally, the Lyapunov exponent are calculated only in the coordinate space. In our case, since high excitation energy may make the phase space explode, we calculate the metric in the whole phase space. The Lyapunov exponent λ are obtained from the relation

$$d(t) = d(0)e^{\lambda t}. \quad (4)$$

In our numerical calculation of Lyapunov exponent we create an initially excited nucleus by the method mentioned above. We call it one test event. At each temperature T, 50 test events are generated. For each test event 50 other events are generated, each of them differing from the test event by the initial $d(t = 0)$. The initial phase distance $d(0)$ for initial systems should be taken as small as possible. In the present case, it is less than 10^{-7}. Let the systems evolve in time based on the QMD model. The exponents are obtained by averaging over 2500 events. In Fig. 2 we plot typical evolutions of $d(t)/d(0)$ at $T = 4$, 12, 25 MeV for nuclei ^{208}Pb. We see that the distance of trajectories increases exponentially with time and can be very well fitted with a straight line (in a semilogarithmic plot), whose slope is just the Lyapunov exponent.

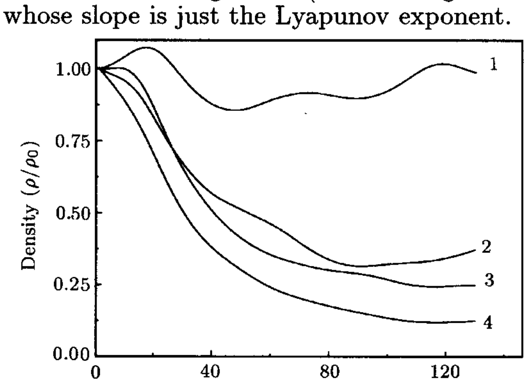

Fig. 1. Time evolution of average density of the nucleus ^{208}Pb at initial temperatures T = 5, 13, 14, 20 MeV (curve 1, 2, 3, 4), respectively.

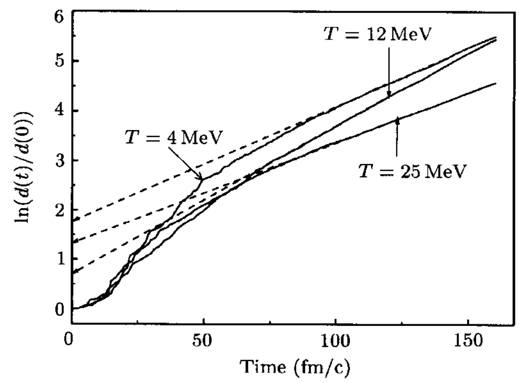

Fig. 2. Logarithmic ratio $d(t)/d(0)$ as a function of time at three initial temperatures T = 4, 12, 25 MeV for the nuclear system ^{208}Pb. The dashed lines are fitted, whose slopes give the typical Lyapunov exponents λ for each temperatures after averaging over 2500 events.

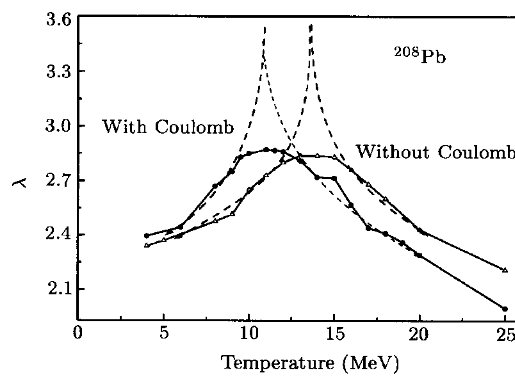

Fig. 3. The largest Lyapunov exponent (in unit of 0.01 c/fm) as a function of temperature in the cases with and without Coulomb term. The dashed lines are fits obtained with the functional form $\lambda = C\,|T - T_c|^{-\omega}$, where $\omega \approx$ 0.13 and C is a constant.

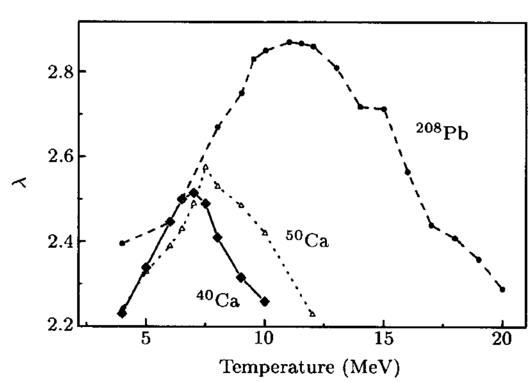

Fig. 4. The largest Lyapunov exponent (in unit of 0.01 c/fm) as a function of temperature for nuclear systems of ^{40}Ca, ^{50}Ca and ^{208}Pb.

Based on the concept of Lyapunov exponent we study the critical phenomenon in expanding process of the initial excited nuclear systems. In Fig. 3 we plot Lyapunov exponents as a function of temperatures for the nucleus ^{208}Pb, where the collision term is switched off. Two curves are for the mean field with and without Coulomb interaction, respectively.

From this figure, we can see that at a low temperature the Lyapunov exponent is smaller and the system still remains at stable. With increasing the initial temperature the exponential divergency of trajectories becomes larger and larger, the chaoticity of the system is getting stronger. At the initial temperature around 11 MeV for the case with Coulomb term and around 13 MeV for the case without Coulomb term the Lyapunov exponent reaches a maximum value, where the fluctuation of the system becomes largest and the chaoticity is strongest. This means that at this temperature the maximum coexistence between the "liguid" and "gas" appears. This seems to be considered as a critical point. Above this initial temperature region, the system is gradually dominated by "gas" (very small fragments or nucleons) and the Lyapunov exponent becomes smaller. In the phase of pure "gas" appeared at higher temperatures, the Lyapunov exponent is getting even smaller. The reason for that is as follows: In a quite high temperature case the interactions within the "gas" is very weak and the exponential divergence in the momentum space almost disappears. Thus, the Lyapunov exponent will decrease. This figure demonstrates the dependence of the Lyapunov exponent on the temperature, which is similar to the relation $\lambda = C\,|T - T_{\rm c}|^{-\omega}$ of the Landau theory of phase transition. In this figure we have also shown this critical expression with $\omega \approx 0.13$ and a constant of C by the dashed lines. Since the system under our consideration is finite, there is, of course, no singularity at the critical point. However, the general trend observed seems to follow the functional form of Landau theory of phase transition. This suggests that the Lyapunov exponent may be considered as one of the good microscopic quantities to describe the critical phenomenon in nuclei.

Based on this study we expect that the interactions may also give a strong influence on the phase transition. By comparing the Lyapunov exponents calculated with and without Coulomb interaction, we can see that Coulomb force makes the critical temperature lower 2 MeV than that for the case without Coulomb term in the system ^{208}Pb. This general tendency is consistent with the case in infinite systems. As we know that for the finite system the influence of the finite size effect of systems on the behavior of phase transition is of great interest. Here we study Lyapunov exponents as a function of temperatures for different systems, for example, ^{40}Ca and ^{50}Ca and so on. In Fig. 4 we show those data together with results of Lyapunov exponents for ^{208}Pb. Thses three curves give similar general behaviors of the phase transition, but the critical temperatures are different. The general tendency is that for the light system the critical temperatute seems to be lower since vaporization of a small system needs less energy. Although this seems to be a trivial conclusion, but for the very finite system it makes a sense, because the most of studies on the liquid-gas phase transition are for infinite systems and are based on the thermodynamic or macroscopic theories. Here we use Lyapunov exponent to measure the critical phenomenon microscopically. We know that the phase transition, in generally speaking, is mainly dependent on the long range interaction. From this point of view we can expect that the collision term should not play an important rule for the phase transition. In Fig. 5 we show the calculated Lyapunov exponents as a function of temperatures for the cases with and without the collision term. This figure tells us the collision term seems to be minor important for our present study on the liquid–gas phase transition in the expanding process of an excited nucleus.

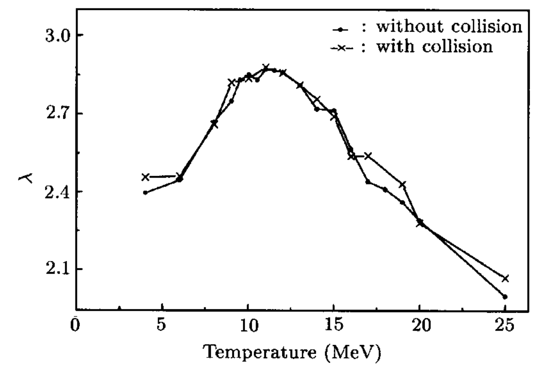

Fig. 5. The largest Lyapunov exponent (in unit of 0.01 c/fm) as a function of temperature at the two cases with and without collision term, respectively.

In summary, we tried to search the microscopic quantity which can be used to describe the phase transition in real nuclei. The Lyapunov exponent is employed and examined for our purpose. The preliminary conclusions are as follows: (1) Lyapunov exponent is a good microscopic quantity to describe the phase transition in nuclear systems, for example, at the later stage of heavy ion reactions. (2) Coulomb potential and the finite size effect may give a strong influence on the critical temperature of the phase transition in hot nuclei. Coulomb force makes the critical temperature lower and increasing the size of systems may enhance the critical temperature. (3) The collision term plays a minor role in the process of the liquid–gas phase transition in finite systems. Finally we would like to mention that there is a considerable difference about the phase transition in the finite and infinite systems. The further study is in progress.

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