# *Non-Gaussian Fluctuation and Non-Markovian Effect in the Nuclear Fusion Process*

—— Langevin Dynamics Emerging from Quantum Molecular Dynamics Simulations

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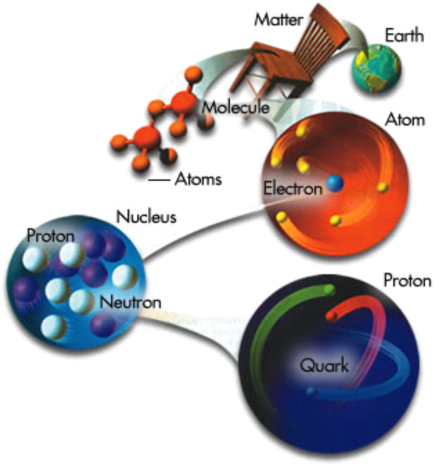
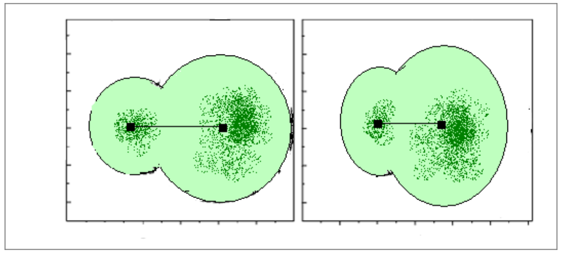
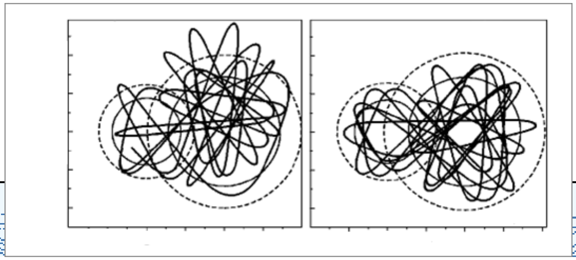


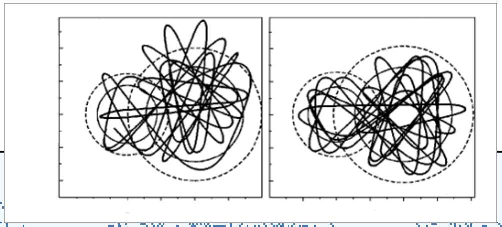


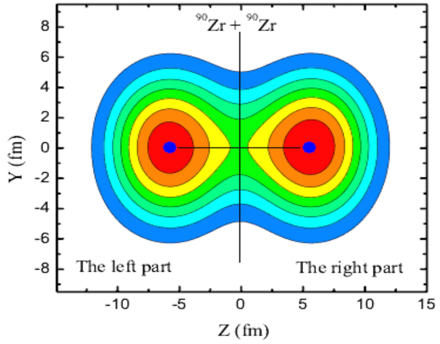
How the macroscopic Langevin-type dynamics emerges out of the microscopic one?

# *In this work*

$$\frac{du(t)}{dt} = -\int_0^t \gamma(t-t')u(t')dt' + \frac{1}{\mu}R(t) - \frac{1}{\mu}\frac{dV(r)}{dr}$$

$^{90}$Zr+$^{90}$Zr





# *Some definition of the variables from Im QMD*

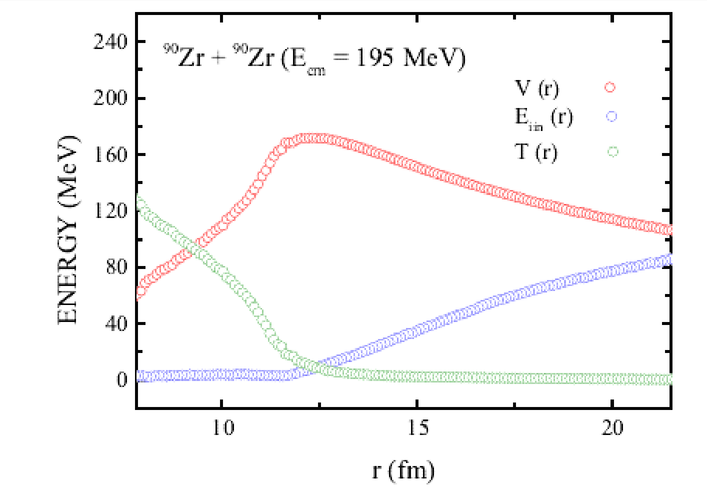
- Collective energy:

$$E_{\text{coll}}(r) = \frac{p^2}{2\mu} + V(r)$$

$$V(r) = E_{\text{tot}}(r) - E_{\text{left}}(r) - E_{\text{right}}(r),$$

- Intrinsic energy:

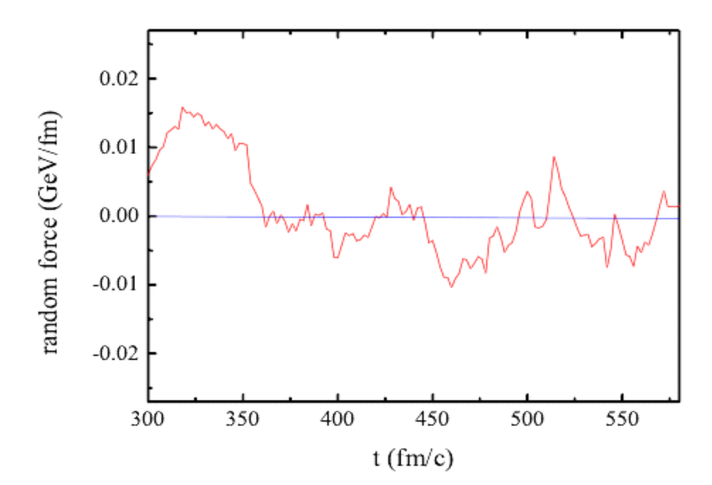
$$E_{\text{intr}}(r) \equiv E_{\text{tot}}(r) - E_{\text{coll}}(r)$$




- Random force in the Langevin equation:

$$R(t)_i \equiv F(t)_i - \langle F(t) \rangle, \quad F(t)_i \equiv \sum_{j=1}^{N} f_i^j(t),$$

$$\langle F(t) \rangle \equiv \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{N} f_i^j(t),$$



# *The development of the distribtution of the random force*

**One may divide the whole**

**process into three regions:**

***Region 1:***

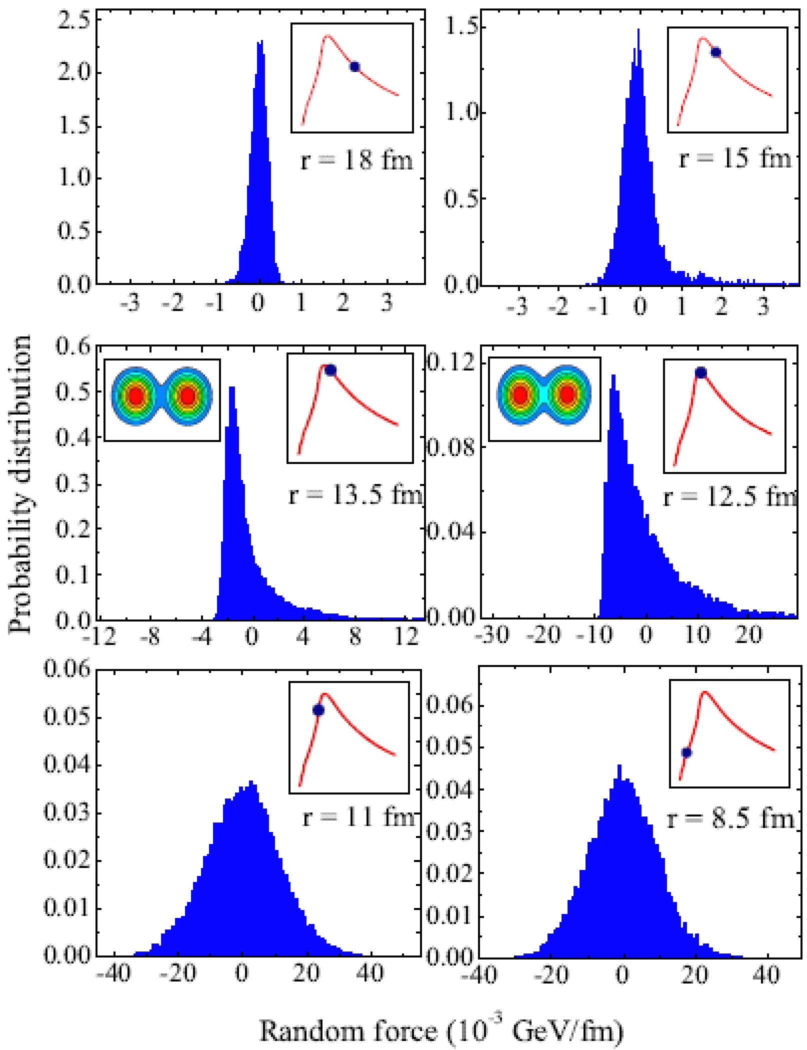
approaching phase up to a touching point where the width of random force has Gaussian form with rather stable and narrow width.

***Region 2:***

from the touching to a barrier top where the width increases rapidly up to a value of almost two order of magnitude larger than that in Region 1, i.e., up to FWHM≈ *1.50×10⁻² GeV/fm.*

***Region 3:***

from the barrier top

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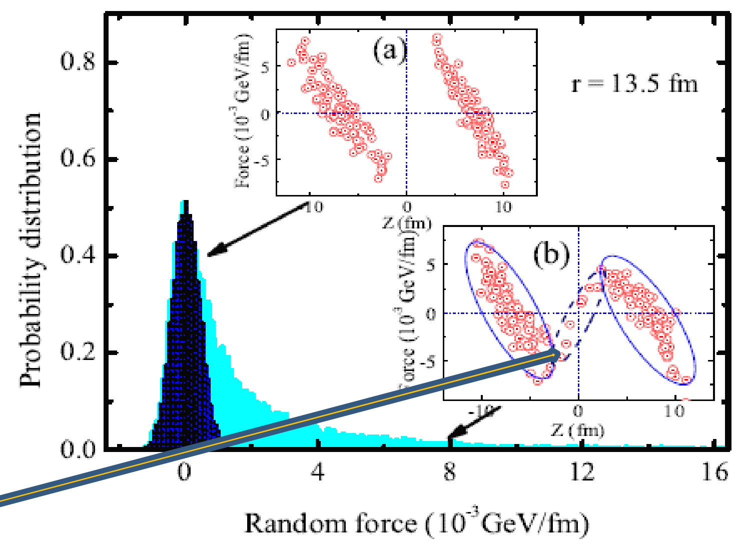
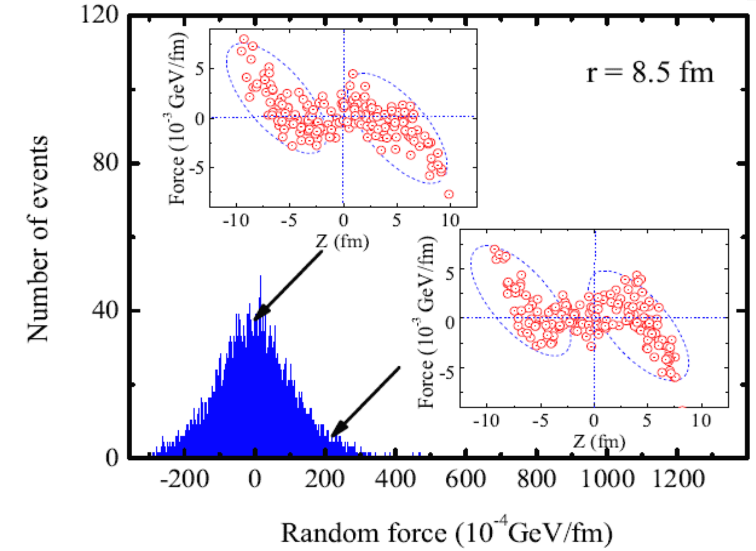
# *Why the non-Gaussian happened in region 2?*

- In region 2, we first divide the non-Gaussian distribution into two parts
  - Fig(a): Events in the symmetric Gauss,
    the whole nucleons are well
    divided into two separated groups so as to keep a stable mean-field.
  - Fig(b): Events in the asymmetric tail.
    a small third group of exchanging nucleon appears.




- In region 3, it becomes very difficult to distinguish the event in the symmetry Gauss from that in the asymmetric tail.

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# *Fluctuation-dissipation relation*

*Fluctuation-dissipation relation links the macroscopic quantity describing the energy dissipation of the collective subsystem to the microscopic characteristic of fluctuations.*

- f [illegible] equation:

$$F = m\frac{du}{dt} = -m\gamma u + R(t)$$ ⇒ $$\langle R(t_1)R(t_2)\rangle = 2m\gamma kT\delta(t_2 - t_1).$$

- From the generalized Langevin equation:

$$\frac{du(t)}{dt} = -\int_0^t \gamma(t-t')u(t')dt' + \frac{1}{\mu}R(t) - \frac{1}{\mu}\frac{dV(r)}{dr}$$ ⇒ $$\langle R(t_1)R(t_2)\rangle = m\gamma kT\gamma(t_2 - t_1).$$

- From the Langevin equation of Mori tpye:

$$\frac{dA}{dt} - i\Omega \cdot A(t) + \int_0^t d\tau\varphi(\tau)\cdot A(t-\tau) = f(t).$$ ⇒ $$\varphi(\tau) \;=\; \langle f(\tau)f^*(0)\rangle \cdot \langle AA^*\rangle^{-1}$$

What kind of relation can we find from the micscopic ImQMD simulation?

# *The fluctuation-dissipation relation extracted from ImQMD*

- **The friction** is calculated assuming the work done by the friction force has completely converted into the

$$\gamma(r) \equiv \frac{\langle F_{\rm fric}(r)\rangle}{\langle p\rangle_r}, \quad F_{\rm fric}(r) \equiv \frac{dE_{\rm intr}(r)}{dr},$$

$$E_{\rm intr}(r) \equiv E_{\rm tot}(r) - E_{\rm coll}(r), E_{\rm coll}(r) = \frac{p^2}{2\mu} + V(r), (4)$$

where

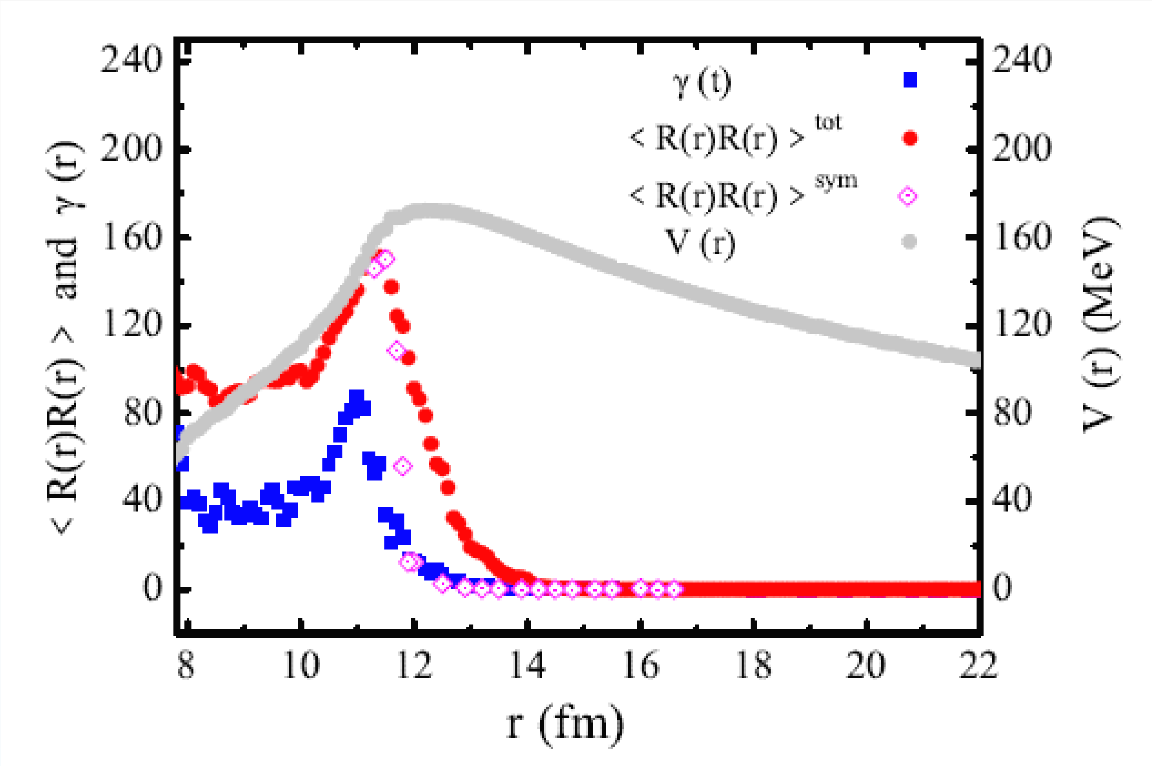
$$\langle p\rangle_r \equiv \frac{1}{n}\sum_i p_i(t_i)|_{\{t_i|r_i(t_i)=r\}},$$

- In order to compare the friction with the **strength of random force** at given relative distance r, we calculate.

$$\langle R(r)R(r)\rangle \equiv \frac{1}{n}\sum_{i=1}^{n} R_i(t_i)R_i(t_i)|_{\{t_i|r_i(t_i)=r\}}$$

# *The fluctuation-dissipation relation extracted from ImQMD*

- Both of the friction and the strength of the random force take almost the same shape and their peaks locate at the same point. The pink diamond is calculated by elimination the asymmetric tail.

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# *The effective temperature*

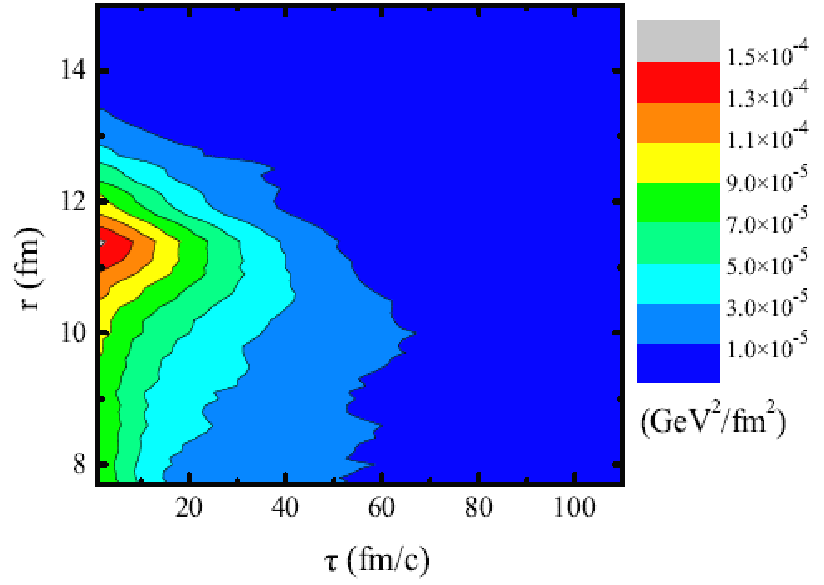
- If Markov approximation is assumed, and a white-noise approximation for the random force:

$$\langle R(t)R(t)\rangle = 2\mu k_B T\gamma \quad \Rightarrow \quad T_{\text{Markov}} = \frac{\langle R(r)R(r)\rangle}{2\mu k_B \gamma(r)}$$

- If Non-Markovian effects is taken, and time correlation of the random force of about 25 fm/c exist (shown by the figure

$$\mu k_B T\gamma(t) = \langle R(0)R(t)\rangle$$

$$T_{\text{non-Markov}} = \frac{\langle p\rangle_r}{\mu k_B}\frac{1}{F_{\text{fric}}(r)} \times \int_0^\infty d\tau \, \frac{1}{n}\sum_{i=1}^{n} R_i(t_i)R_i(t_i-\tau)\Big|_{\{t=i|r(t_i)=r\}}$$

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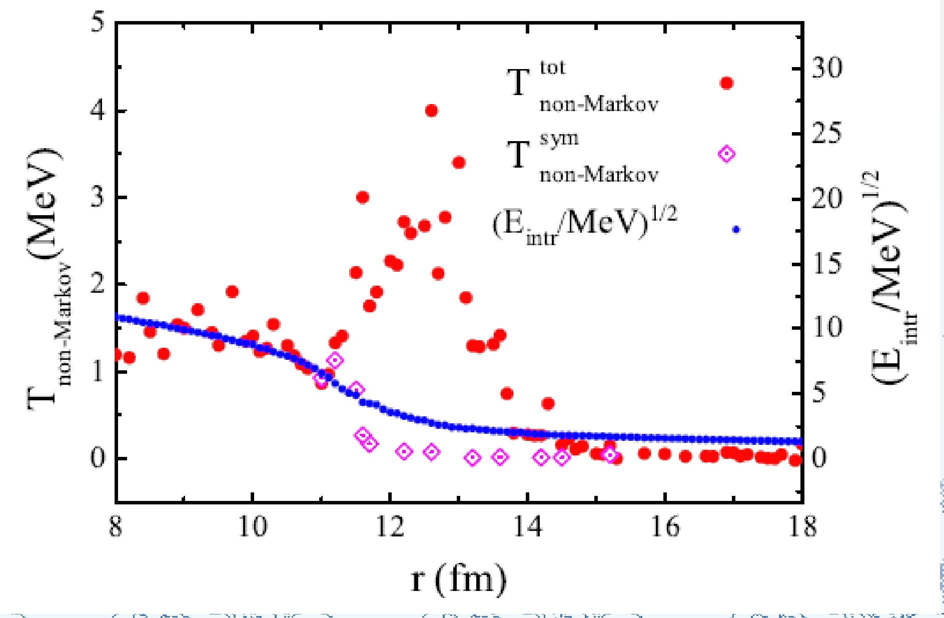
# *The effective temperature*

- *Macro dynamics of relative motion described by the one-dimensional Langevin is not appropriate in Region 2.*
- *T* $\sqrt{E_{intr}}$ *ffective temperature* $T_{non\text{-}Marcov}$ *shows consistent feature with*

which represents the temperature under the Fermi gas mode.

- $T_{ma}$

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# Conclusions

- The dissipation dynamics of the relative motion between two fusing nuclei is associated with non-Gaussian distributions of the random force.
- Microscopic information of the random force as well as of its time correlation function are analyzed.
- A proper treatment of the non-Markovian (memory) effect in the Langevin dynamics are decisive for the dynamics of emergence in the nuclear dissipative fusion motion.