

# Overview of nuclear bremsstrahlung and proposal of proton/nucleus-nucleus experiments at IMP

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# Outline:

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1. Motivations to study bremsstrahlung in proton-nucleus interactions.
2. Progress in study photons in nuclear reactions: experiment & theory
3. Proposals for possible new experiments
4. Symposium on Physics of Photons at IMP

# Why to study bremsstrahlung?

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## **Motivations:**

- 1) To know nuclear interactions in particles/nuclei collisions;
- 2) To know internal structure of particles, participating in reaction;
- 3) To know dynamics of collisions;
- 4) To know properties of quark matter in p-nucleus collisions;
- 5) To know properties of tunneling.

## **Studied reactions (experiments):**

- Collisions of protons with nuclei;
- Collisions of alpha-particles with nuclei;
- Collisions of protons (deuterons, alpha-particles) with protons (deuterons, alpha-particles);
- Collisions of light nuclei (ions) with nuclei;
- Alpha-decay, proton-decay, cluster-decay of heavy nuclei;
- Spontaneous fission, ternary fission,
- Fusion of superheavy nuclei.

# Motivations

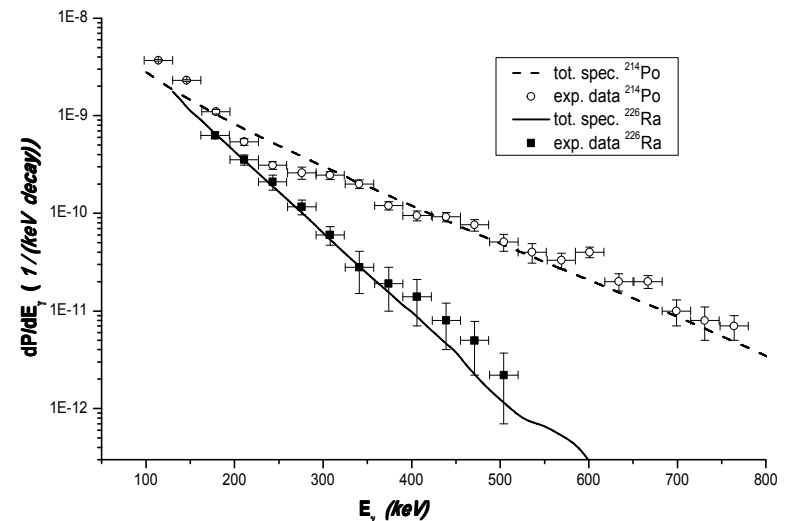
- Bremsstrahlung photons give more rich information about studied nuclear process, than direct study of this process.

*Example:* Alpha-decay of nucleus

1) Direct study gives two parameters: Q-value, half-live  $T_{1/2}$ .

2) Study of bremsstrahlung photons gives infinite number of parameters:

- Spectra (probabilities at different energies of emitted photons);
- Spectra at different angles,
- For deformed nucleus - at different orientations of this nucleus concerning directions of emitted photon and moving alpha-particle.



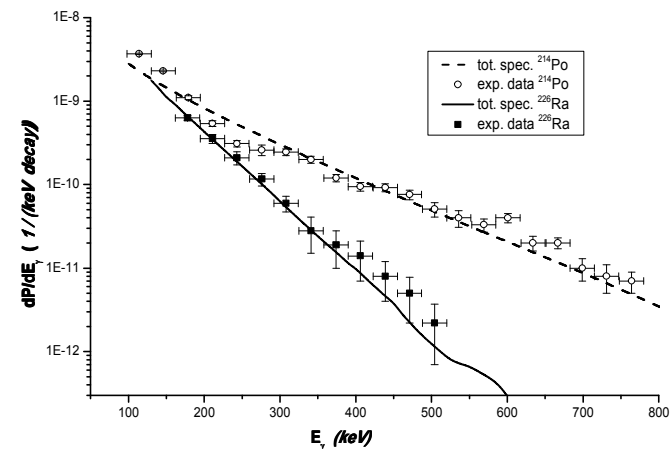
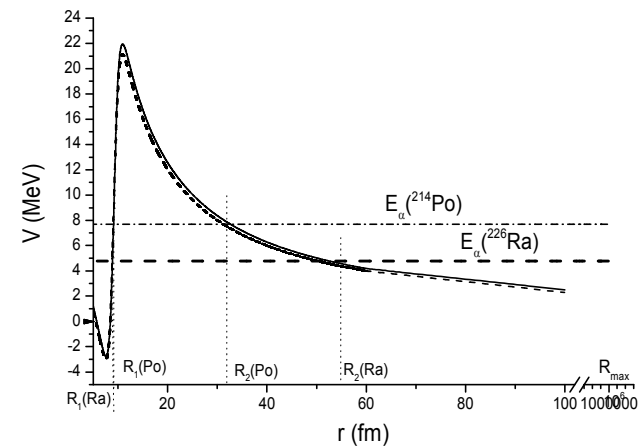
# Motivations

- Bremsstrahlung spectrum is essentially more sensitive to interacting potential of nuclear process, than direct task of this process.

## *Example:* Alpha-decay of nucleus

- 1) Direct study: half-lives are sensitive to potential shape up to 100 fm.
- 2) Study of bremsstrahlung photons :
  - Spectra are sensitive to potential shape up to atomic shells,
  - Shape of potential in tunneling region plays the most important role.

*Higher sensitivity is key idea, how we study quarks in proton-nucleus collisions*



# Experiments:

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Study of bremsstrahlung emission of photons in:

- Scattering of protons on nuclei:

- $E_{\text{proton}} = 27 \text{ MeV}$ ,  $E_{\text{photon}} = 7\text{-}35 \text{ MeV}$ , target =  $^{197}\text{Au}$  [1];
- $E_{\text{proton}} = 72 \text{ MeV}$ ,  $E_{\text{photon}} = 22\text{-}74 \text{ MeV}$ , target =  $^{12}\text{C}$ ,  $^{64}\text{Cu}$ ,  $^{107}\text{Ag}$ ,  $^{197}\text{Au}$  [2];
- $E_{\text{proton}} = 140\text{MeV}$ ,  $E_{\text{photon}} = 22\text{-}120\text{MeV}$ , target = H, D, Be, C, N, O, Al, Cu, Pb[3].

[1] D.R.Chakrabarty et al., Phys. Rev. **C60**, 024606 (1999).

[2] A.Kwato Njock et al., Phys. Lett. **B207**, 269 (1988).

[3] J.A.Edington, B.Rose, Nucl. Phys. **89**, 523 (1966).

- Scattering of light nuclei on heavy nuclei:

- $E_{\text{beam}}/A = 5\text{-}12 \text{ MeV/nucleon}$ ,  $E_{\text{photon}} = 5\text{-}28 \text{ MeV}$ ;
- $E_{\text{beam}}/A > 20 \text{ MeV/nucleon}$ ,  $E_{\text{photon}} > 25 \text{ MeV}$ .

- Collisions of protons & neutrons between themselves:

- $E_{\text{beam}} = 310 \text{ MeV}$  [4],  $E_{\text{beam}} = 200 \text{ MeV}$  [5],

[4] A.Johansson et al., Phys. Rev. **C83**, 054001 (2011).

[5] J.G.Rogers et al., Phys. Rev. **C22**, 2512 (1980).

# Main models:

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- **Boltzmann-Uehling-Uhlenbeck (BUU) transport model:**
  - [1] Bertsch et al., PRC **29**, 675 (1984), PRC **31**, 1730 (1985), PRC **34**, 2127 (1986), etc.
  - [2] W.Cassing et al., Phys.Rep. **188**, 363 (1990).
  - [3] C.B.Das et al., Phys.Rev. **C67**, 034611 (2003).
  - [4] Liu Gui-Hua et al., Phys. Lett. **B663**, 312 (2008), Chin. Phys. **C33**, 89 (2009).
  - [5] Gao-Chan Yong et al., Phys Let. **B705**, 240 (2011).
- **Relativistic gauge invariant nucleon-nucleon approach:**
  - [1] K.Namayama et al., PRC **85**, 064001 (2012), PRC **83**, 054001 (2011), PRC **80**, 051001 (2009), PRC **52**, 2377 (1995), PRC **43**, 394 (1991), etc.
  - [2] M.K.Liou et al., PRC **54**, 1574 (1996), etc.
- **Quantum model with realistic p-nucleus potential:**
  - I.V.Kopitin et. al., Yad. Fiz. **60**, 869 (1997);
  - S.P.Maydanyuk, V.S.Olkhovsky + G.Giardina et al. + N.Eremin et al.: PTP 109, 203 (2003), EPJA 28, 283 (2006), EPJA 36, 31 (2008), MPLA 23, 2651 (2008), NPA 823, 38 (2009), IJMPE 19, 1189 (2010), PRC 82, 014602 (2010); JPG 38, 085106 (2011), PRC 86, 014618 (2012).

# Main models: comparative analysis

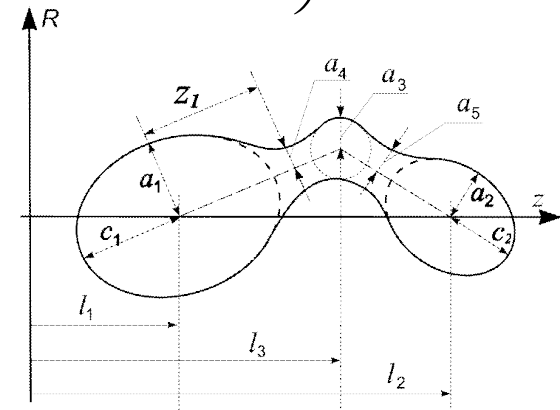
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BUU transport model VS quantum potential model (QPM):

1. QPM is based on many nucleons generalization of Dirac equation:

$$i\frac{\partial\psi}{\partial t} = \left( \sum_{i=1}^{A+1} \boldsymbol{\alpha} \left( \mathbf{p}_i - \frac{z_i e}{c} \mathbf{A}_i \right) + \sum_{i=1}^{A+1} (z_i e A_{i,0} + \beta m_i) + V(\mathbf{r}_1 \dots \mathbf{r}_{A+1}) \right) \psi$$

2. In QPM, potential is calculated on the basis of continuous deformation of shape of full nuclear system



3. QPM uses fully quantum approach for calculations of wave functions, role of boundary conditions



# Main models: comparative analysis

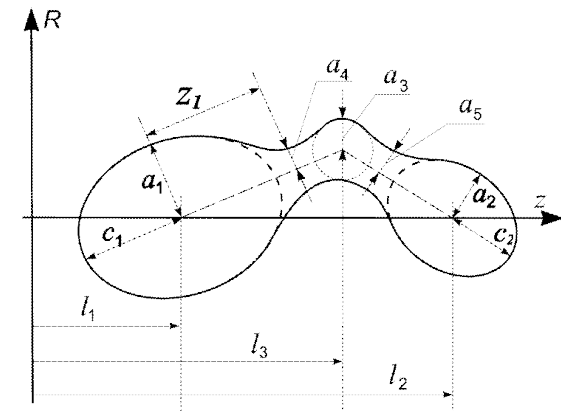
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Relativistic gauge invariant nucleon-nucleon approach  
VS quantum potential model (QPM):

1. QPM describes many-nucleons interactions (*which are not small!*)

$$\hat{H} \varphi = E \varphi, \quad \hat{H} = \sum_{i=1}^{A+1} \left\{ \frac{1}{2m_i} \left( \mathbf{p}_i - \frac{z_i e}{c} \mathbf{A}_i \right)^2 + z_i e A_{i,0} - \frac{z_i e \hbar}{2m_i c} \boldsymbol{\sigma} \cdot \text{rot}_i \mathbf{A}_i \right\} + V(\mathbf{r}_1 \dots \mathbf{r}_{A+1}).$$

2. QPM determines potential using shape of full nuclear system (collective dynamics is present)



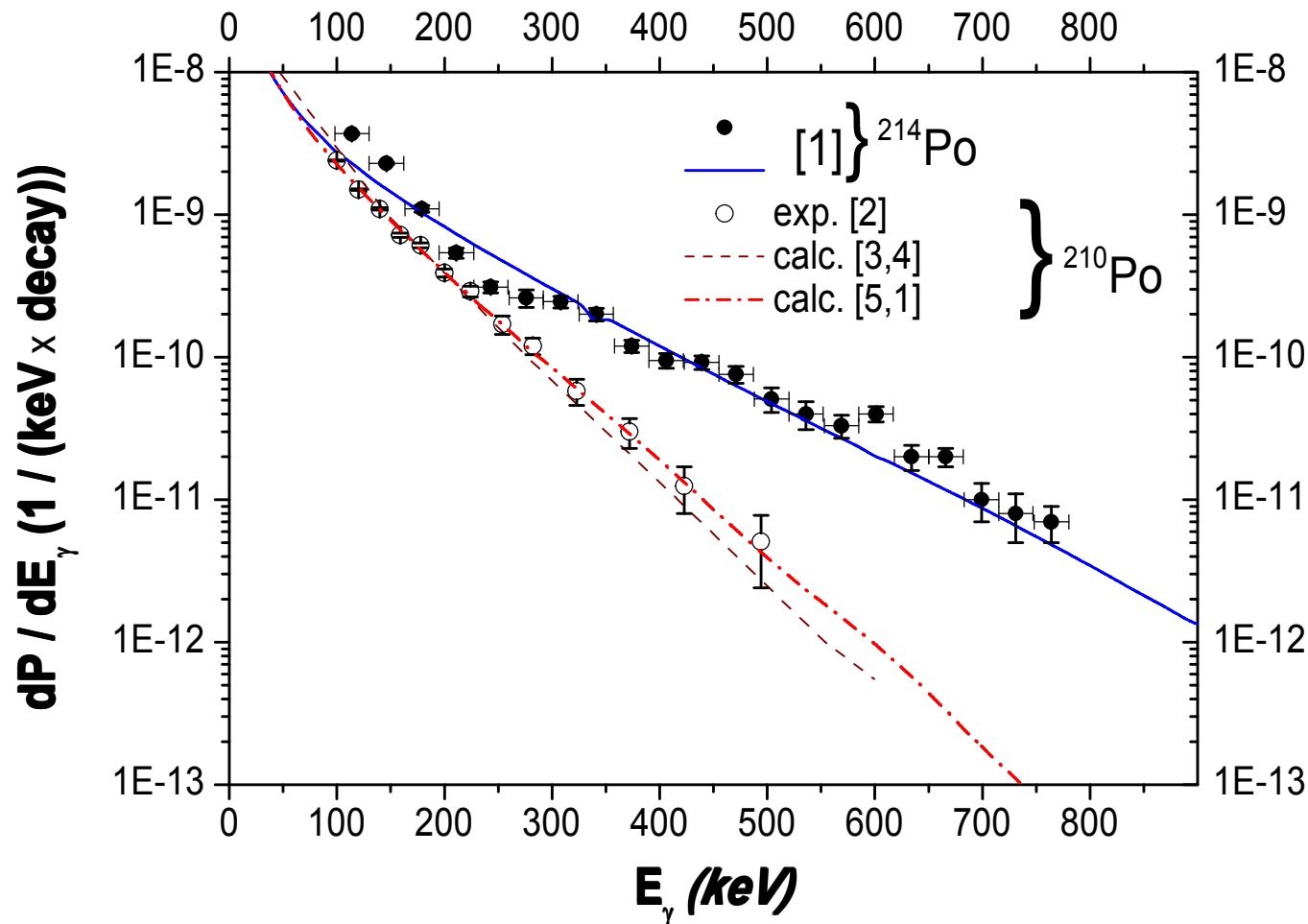
# Our model

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## **Sketch:**

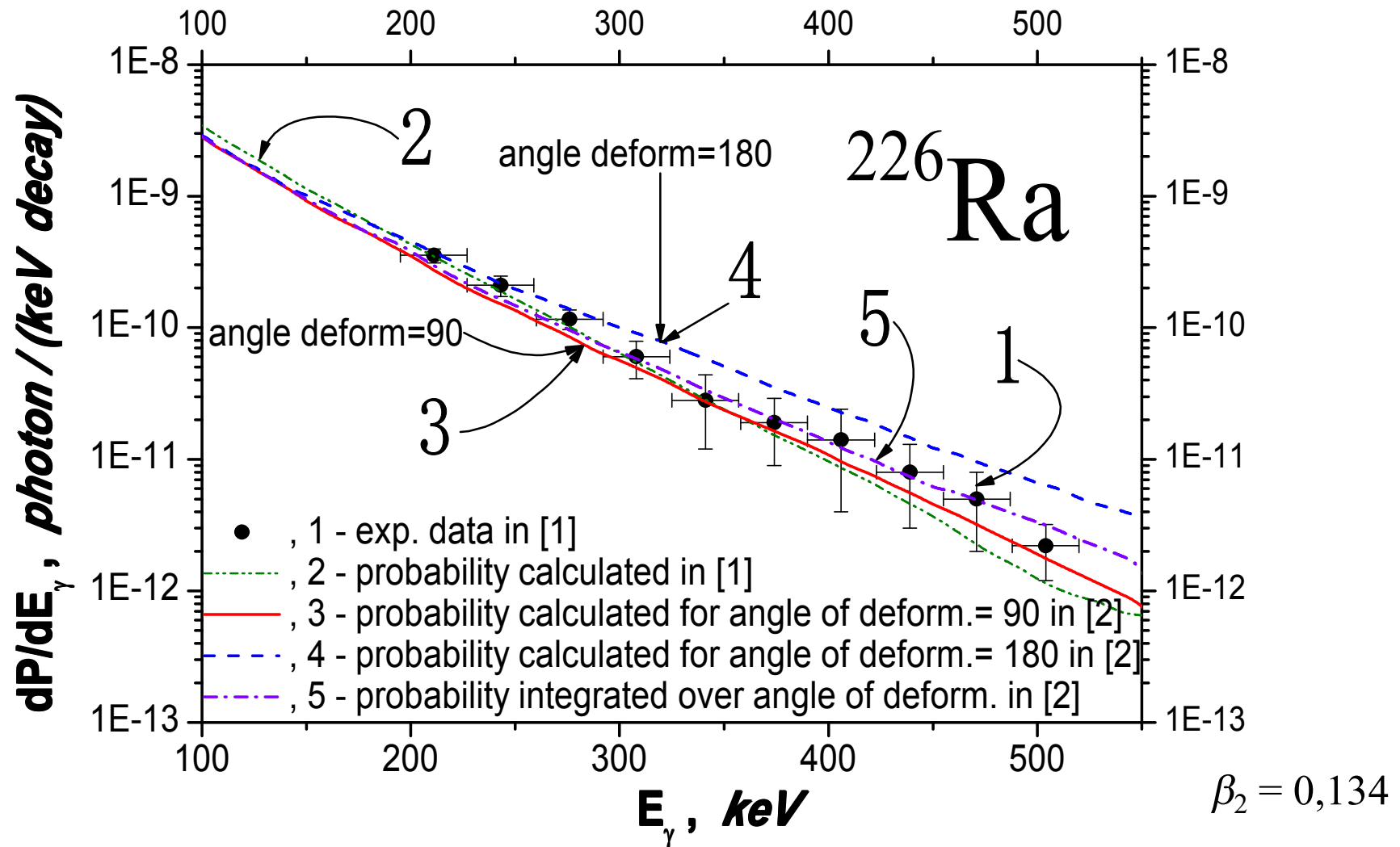
1. Two nucleons interactions are based on Dirac equation
2. Nuclear system – full set of interacting nucleons
3. Surface of nuclear system is changed dynamically
4. Potential of interaction between fragments (protons, nuclei) – tested by experimental data
5. Wave functions, boundary conditions
6. Bremsstrahlung probabilities
7. Calculations of spectra: for light and heavy masses of fragment, from zero up to high energies of photons

# Bremsstrahlung for $\alpha$ -decay of $^{210}\text{Po}$ , $^{214}\text{Po}$



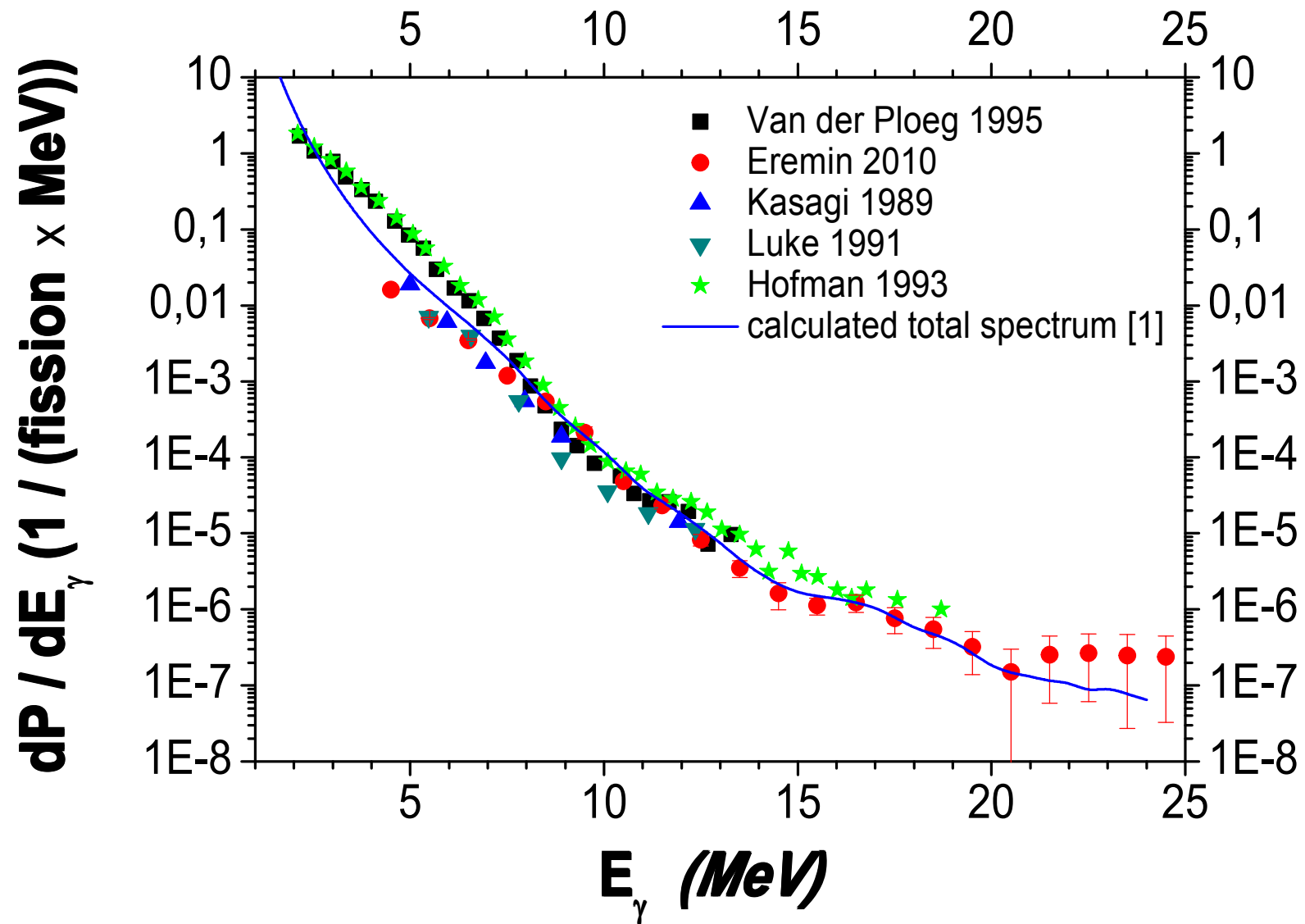
1. G. Giardina et al., Europ. Phys. Journ. **A36**, 31-36 (2008).
2. G. H. Boie, H. Scheit, U. D. Jentschura et al., Phys. Rev. Lett. **99**, 022505 (2007).
3. U. D. Jentschura, A. I. Milstein, I. S. Terekhov et al., Phys. Rev. **C77**, 014611 (2008).
4. G. Th. Papenbrock, G. F. Bertsch, Phys. Rev. Lett. **80**, 4141 (1998).
5. S. P. Maydanyuk, V. S. Olkhovsky, Europ. Phys. Journ. **A28**, 283-294 (2006).

# Photons in $\alpha$ -decay of deformed $^{226}\text{Ra}$



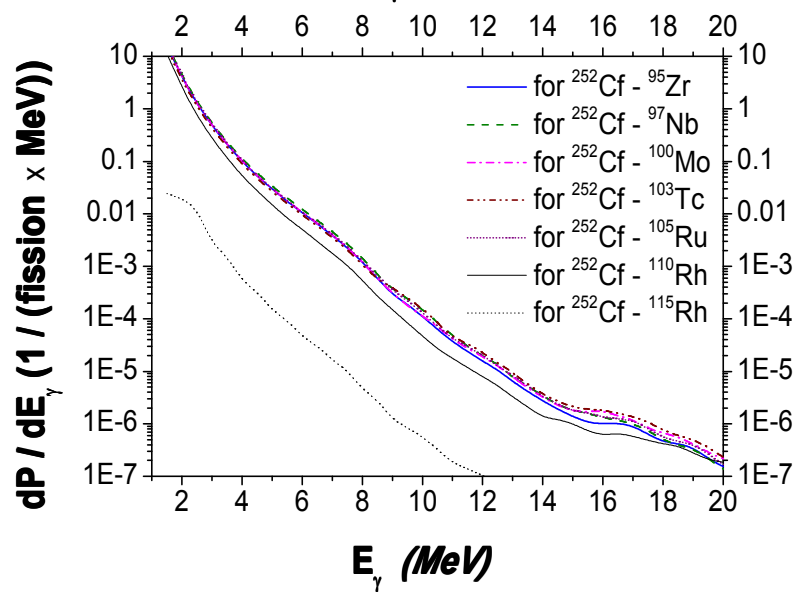
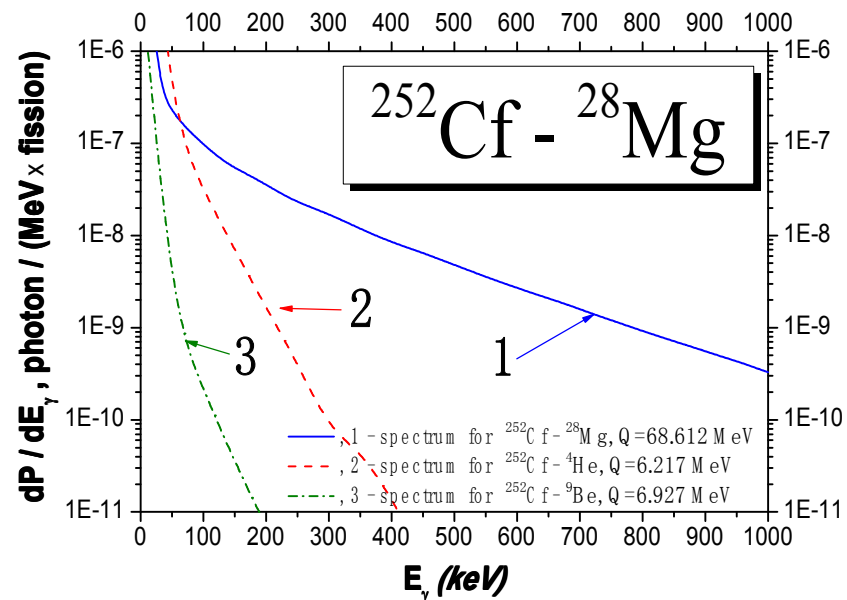
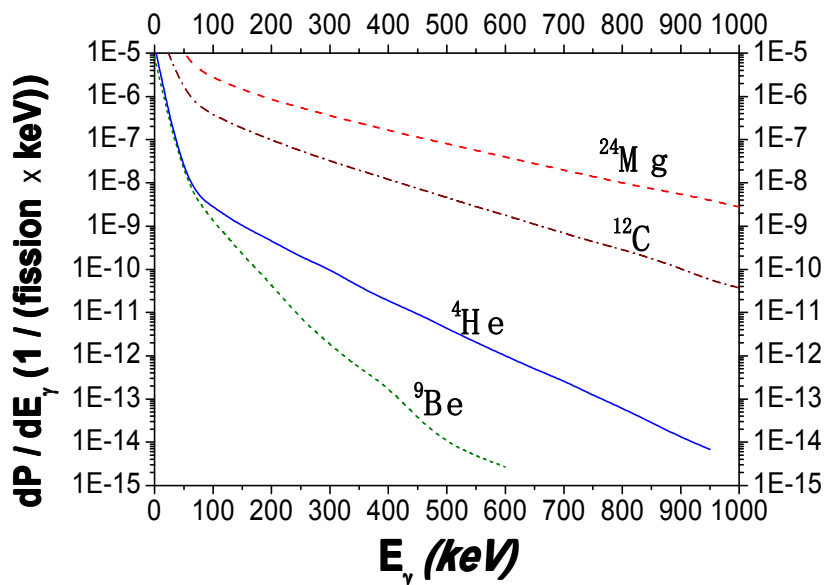
1. G. Giardina, G. Fazio, G. Mandaglio et al., Mod. Phys. Lett. **A23**, 2651 (2008).
2. S. P. Maydanyuk, V. S. Olkhovsky G. Giardina et al., Nucl. Phys. **A823**, 38 (2009).

# Bremsstrahlung in fission of $^{252}\text{Cf}$



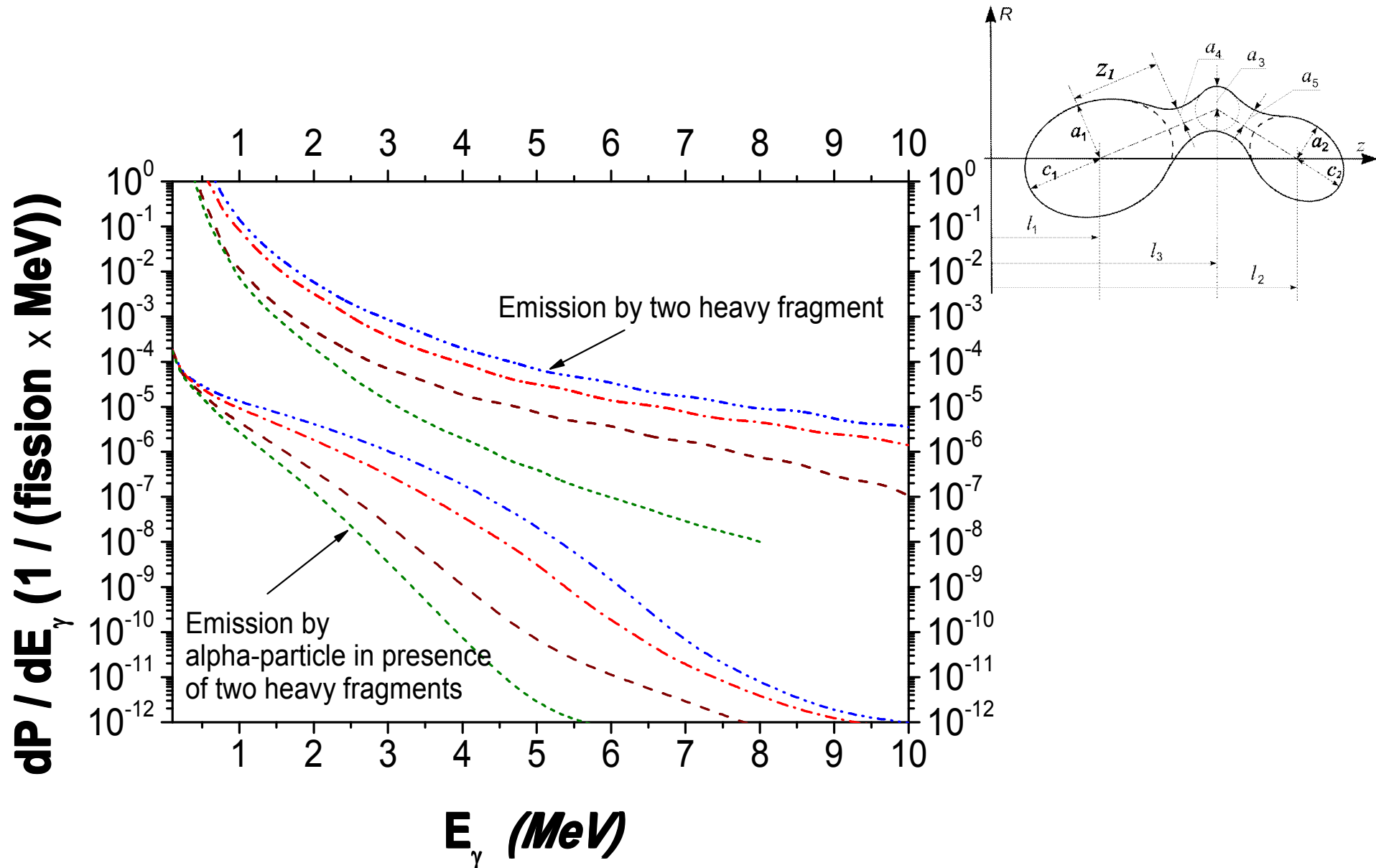
1. S.P.Maydanyuk, V.S.Olkhovsky, G.Mandaglio et al., Phys. Rev. **C82**, 014602 (2010).

# Fission: Emission of photons by fragments



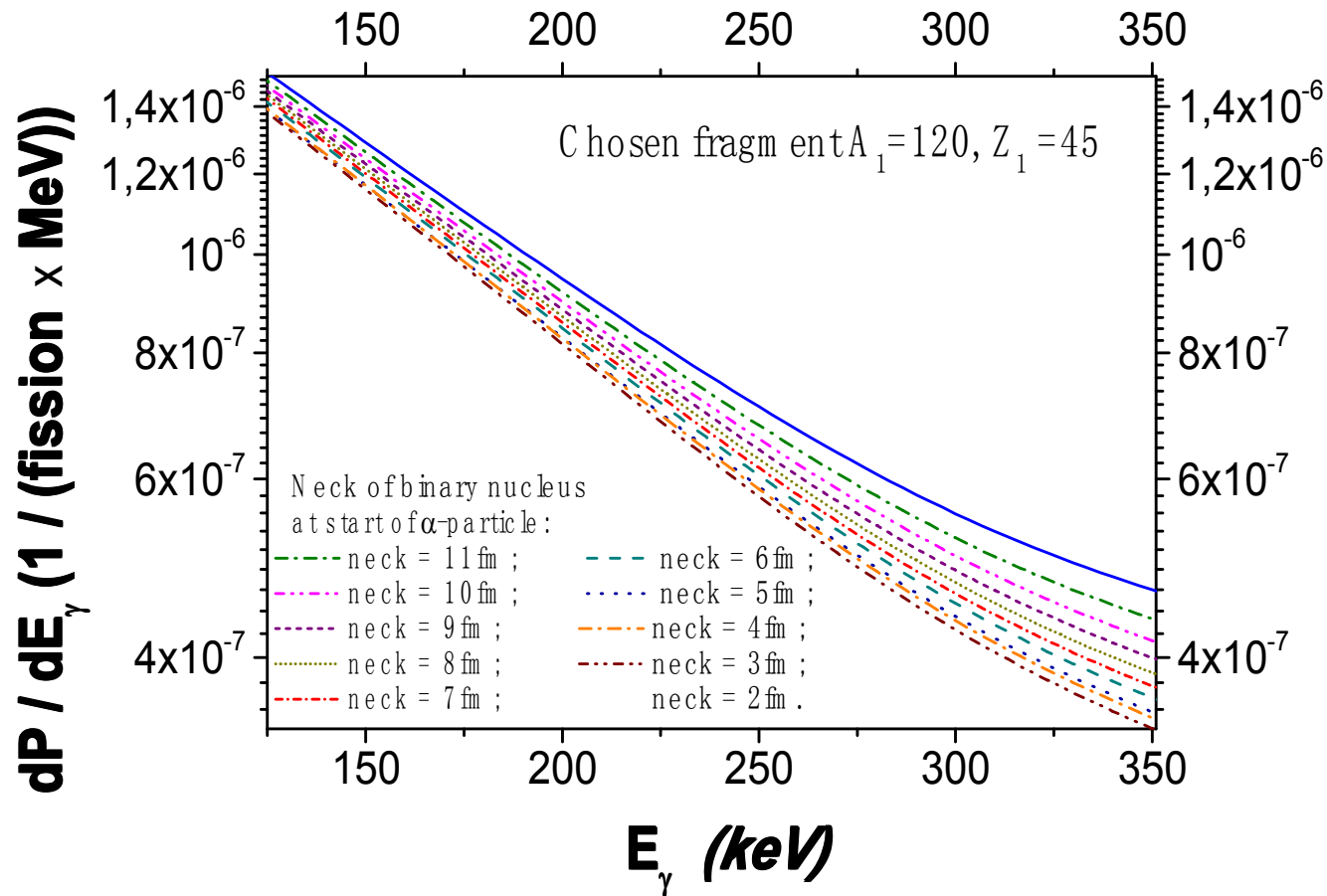
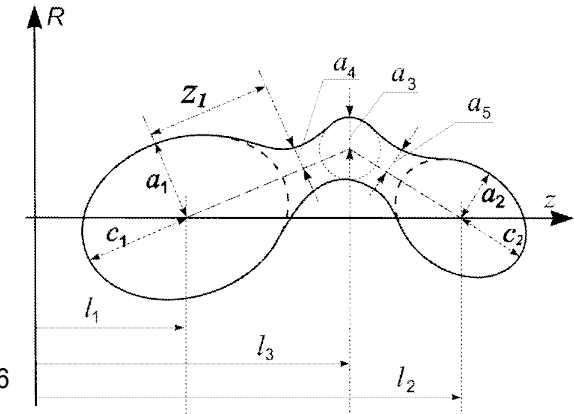
# Ternary fission: Contribution of binary nucleus

**Analysis**



# Role of neck at start of $\alpha$ -particle

If binary nucleus gives own contribution at high energies (above 300keV), then at low energies we have emission by  $\alpha$ -particle only.





# Proton-nucleus interactions

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## At low energy of emitted photon (up to 500 keV):

Bremsstrahlung during decay of nucleus with emission of proton.

## At intermediate energy of emitted photon (20 - 120 MeV):

Bremsstrahlung during collisions of proton off nucleus.

**Experiments:** [1]  $p + {}^9\text{Be}$ ,  $p + {}^{12}\text{C}$ ,  $p + {}^{208}\text{Pb}$ ,  $E_p = 140 \text{ MeV}$ ,

[2]  $p + {}^{12}\text{C}$ ,  $p + {}^{64}\text{Cu}$ ,  $p + {}^{107}\text{Ag}$ ,  $E_p = 72 \text{ MeV}$ .

## Questions:

How intensive is the magnetic emission?

Angular analysis of emission;

How is emission changed in dependence on distance between proton and nucleus?

How intensive is the magnetic emission in the tunneling region?

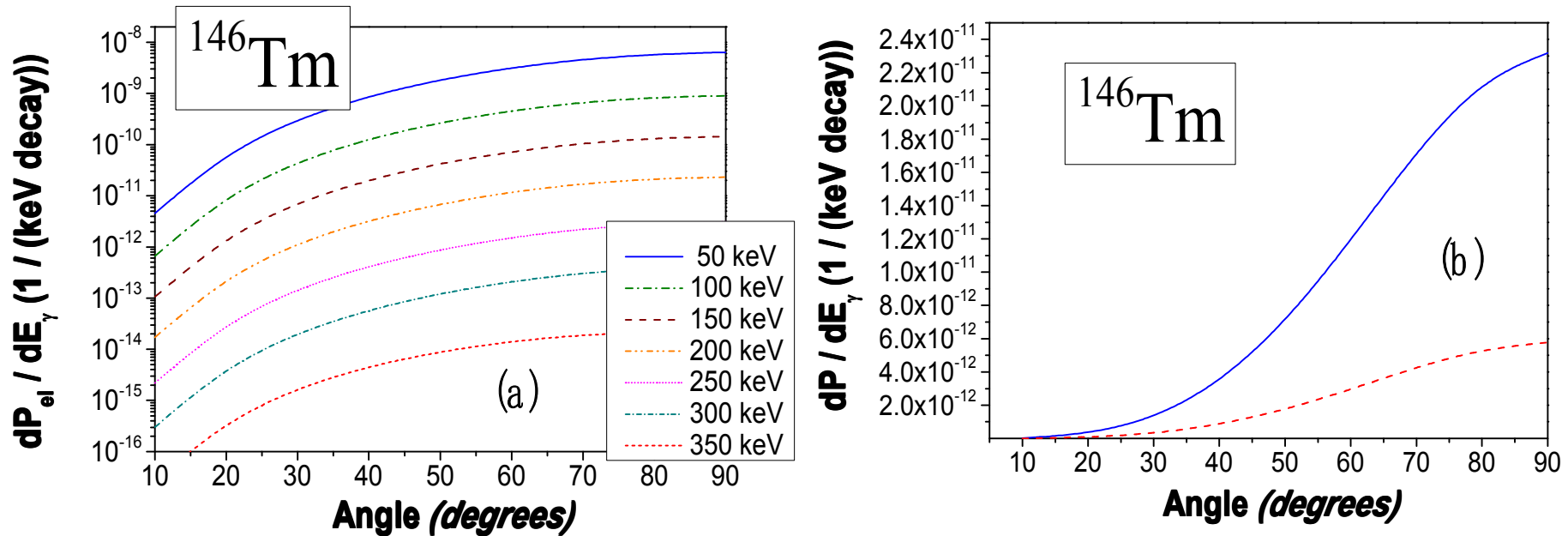
How are the spectra changed close to zero energy limit of the emitted photons?

Which values has probability at maximum and at zero energy limit of photons?

[1] J.Edington, B.Rose, Nucl. Phys. **89**, 523 (1966).

[2] M.Kwato Njock et al., Phys. Lett. **B207**, 269 (1988).

# Bremsstrahlung in proton decay of $^{146}\text{Tm}$ : *angular analysis*



Angular distributions of the bremsstrahlung emission during proton decay:

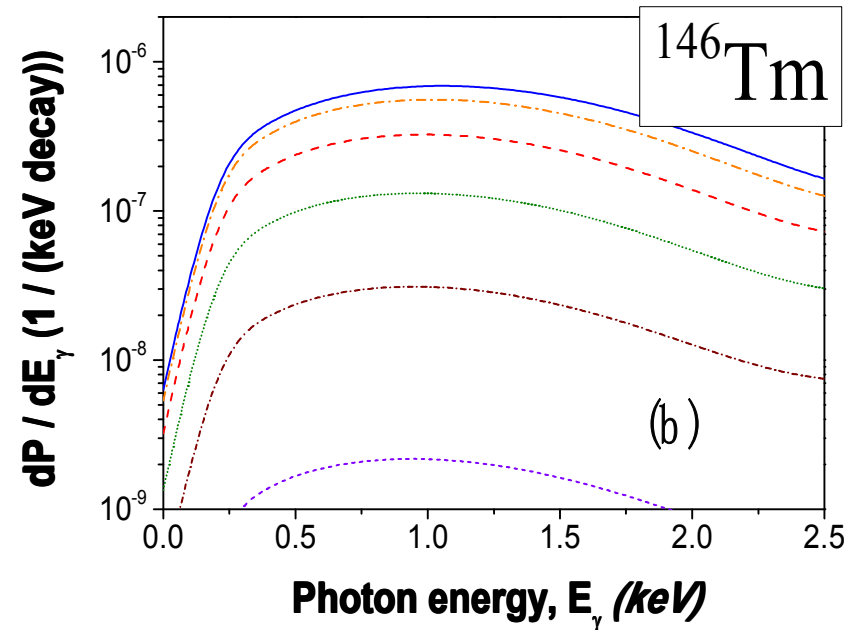
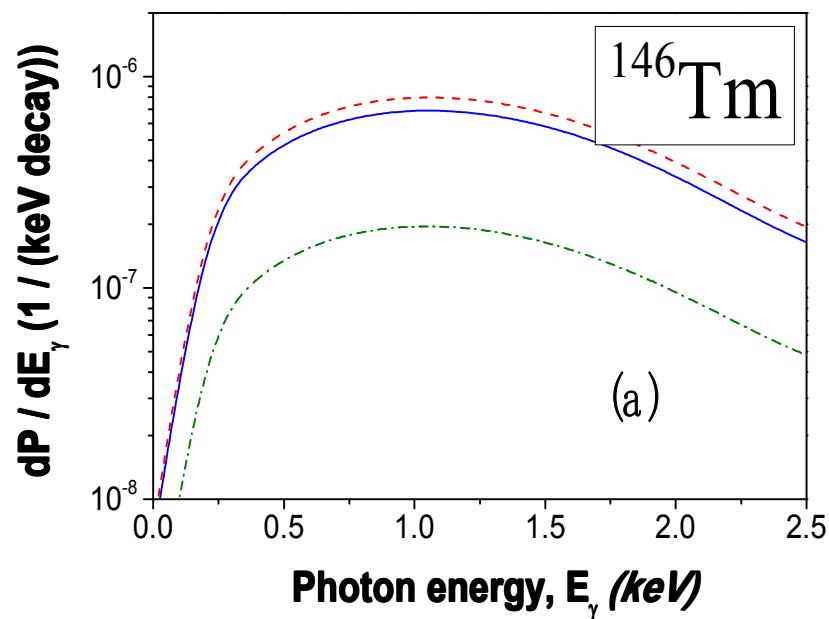
(a) the electric component of emission,  $dP_{el}$ , calculated at different energies of the emitted photons;

(b) the electric component  $dP_{el}$  (full blue line) and magnetic component  $dP_{mag,1}$  (red dashed line) for the chosen photon energy 200 keV. One can see that both spectra increase proportionally with increasing of the angle.

# Bremsstrahlung in proton decay of $^{146}\text{Tm}$ : *emission of soft photons*

According to QED theory, divergence in calculation of matrix element is appeared at limit of photon energy to zero. Here, two prevailing approaches are known:

- 1) soft-photon theorem to all nuclear bremsstrahlung processes (Low);
- 2) approximation of Feshbach and Yennie.

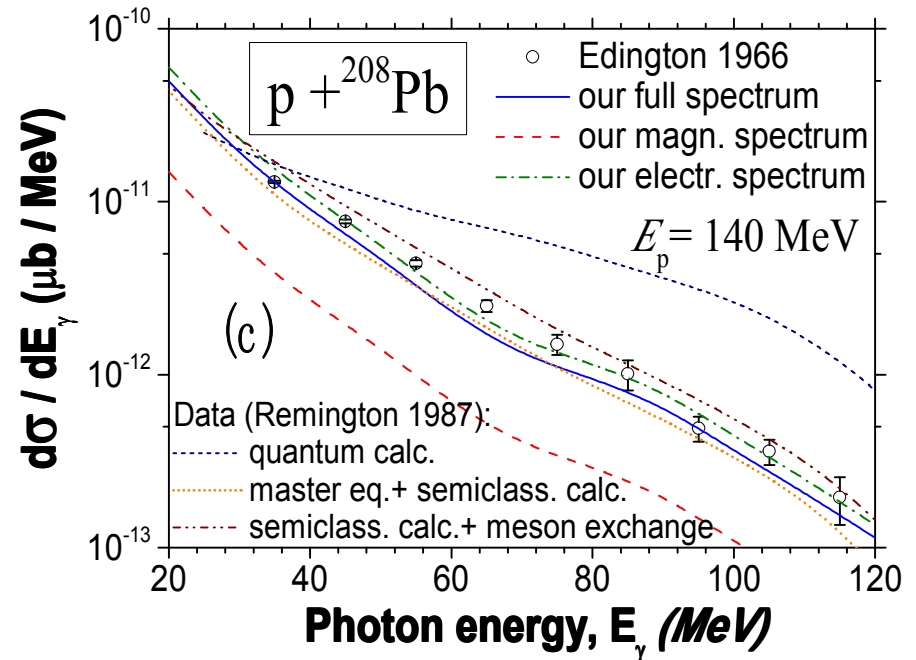
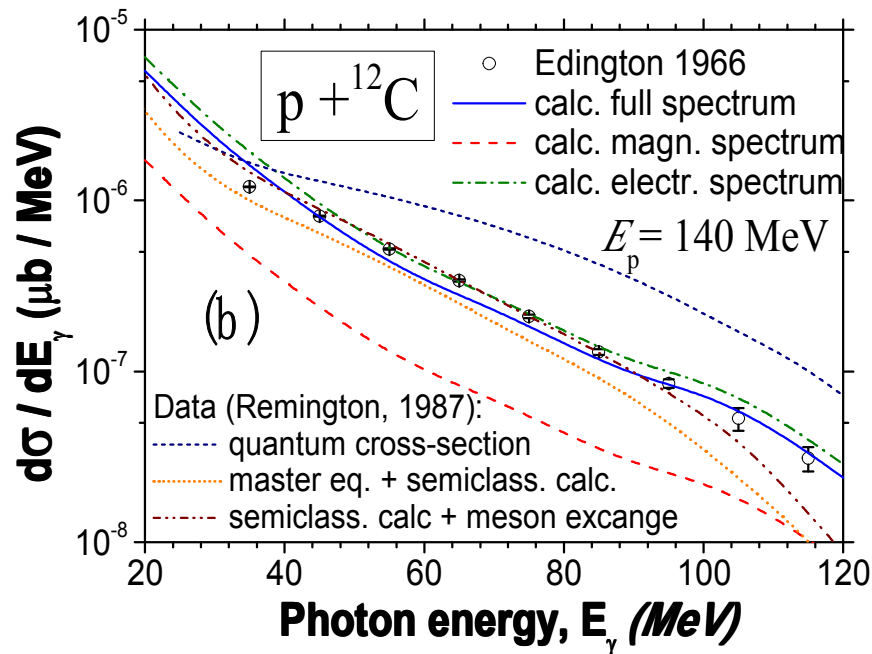


In Fig. results in our potential approach: (a) full spectrum (full blue line), electric component (red dashed line), magnetic component (green dash-dotted line) ( $\theta=90^\circ$ ); (b) full spectrum in dependence on angle  $\theta$  (full blue line for  $\theta=90^\circ$ , orange dash-dotted line for  $\theta=75^\circ$ , red dashed line for  $\theta=60^\circ$ , olive short dotted line for  $\theta=45^\circ$ , wine short dash-dotted line for  $\theta=30^\circ$ , violet short dashed line for  $\theta=15^\circ$ ).

***So, there is no the infrared catastrophe in our approach.***

# Bremsstrahlung in proton nucleus collisions: incident energy $T_{\text{lab}} = 140 \text{ MeV}$

Comparison for  $p + {}^{12}\text{C}$  and  $p + {}^{208}\text{Pb}$  between calculations by our model [1], calculations in [2], and experimental data [3].



- [1] Maydanyuk, Phys. Rev. **C86**, 014618 (2012): blue solid line is for full spectrum, green dash-dotted line for electric contribution, red dashed line for magnetic contribution.
- [2] Remington 1987: B.A.Remington, M.Blann, G.F.Bertsch, Phys. Rev. **C35**, 1720 (1987). wine dash double-dotted line is for calculations by master equation using the semiclassical bremsstrahlung cross sections, orange short dotted line for semiclassical cross sections multiplied by 2 for meson exchange, and navy short dashed line for quantum bremsstrahlung cross sections.
- [3] Edington 1966: J.Edington, B.Rose, Nucl. Phys. **89**, 523 (1966).

# Joint project

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## *Scheme of joint group*

International group:

- S.P.Maydanyuk, V.S.Olkhovsky [1],
- G.Giardina, G.Mandaglio, PhD students [2]

- Theory, calculations, predictions [1]
- Analysis of experimental data [2]

Chinese groups:

IMP group,  
SINAP group,

***Other laboratories  
welcome!***

- We have well working theory.
- We need in experimental support, any experimental news.

[1] Institute for Nuclear Research, Kiev, National Academy of Science of Ukraine,  
[2] Dipartimento di Fisica dell'Universita di Messina, and  
Istituto Nazionale di Fisica Nucleare, Sezione di Catania, Italy.

# Proposal

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**Main aim:** to develop high sensitive tools for study of properties and dynamic of nuclear and quark matters on the basis of bremsstrahlung photons analysis

## Motivations:

- Experimentalists wrote about crucial importance of any new experimental information [1-2]
- Photons in some types of reactions have not been measured (any new information here will be the first)
- Development of theory with such experiments allows to construct tools with higher sensitivity *(there is no need to reach huge energies and construct expansive experimental facilities for study of quarks properties!)*

[1] A.Kwato Njock et al., Phys. Lett. **B207**, 269 (1988).

[2] V.V.Kamanin et al., Phys. El. Part. At. Nucl. **20**, 743 (1989).

# Future research

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- Scattering of protons on nuclei:

- To re-measure spectrum for  $^{64}\text{Cu}$  ( $E_{\text{proton}} = 72 \text{ MeV}$ ,  $E_{\text{photon}} = 20\text{-}74 \text{ MeV}$  [1]);
- To measure spectra for strongly deformed nuclei;
- To measure angular distribution of photons (if possible);
- To formulate new method of determination of p-nucleus potentials;
- To find any non-monotonic shape of spectra at high accuracy (not long range);
- To measure spectra at smallest energies of photons:
  - We predict maximal limit of probability and absence of infrared catastrophe in QED,
  - Corrections of approximation of Feshbach and Yennie [2], soft-photon theorem [3];

[1] A.Kwato Njock et al., Phys. Lett. **B207**, 269 (1988).

[2] H.Feshbach, D.R.Yennie, Nucl. Phys. **37**, 150 (1962).

[3] F.E.Low, Phys. Rev. **110**, 974 (1958).

- Scattering of light nuclei on heavy nuclei:

- To measure spectra outside kinematic energy limit  $E_{\text{beam}}/A$ ;
- To clarify EMC effect;

- Synthesis of superheavy nuclei: everything (this will be the first information)

# Proposal: Photons in scattering of proton on strongly deformed nucleus

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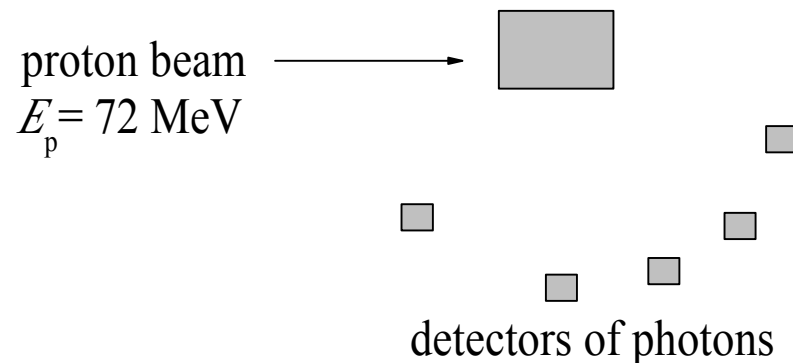
Aim: To construct new experimental tools to measure deformation of nucleus.

Target: nuclei with large quadrupole deformation.

Incident proton energy  $T_{\text{lab}} = 72 \text{ MeV}$ ,

Energy region of measured photons  $E_{\text{ph}} = 0-5, 10-75 \text{ MeV}$ ,

To measure spectra at angles 15, 30, 45, 75, 90, 135.





# Proposal: Photons in scattering of proton on nucleus at high photons energy limit

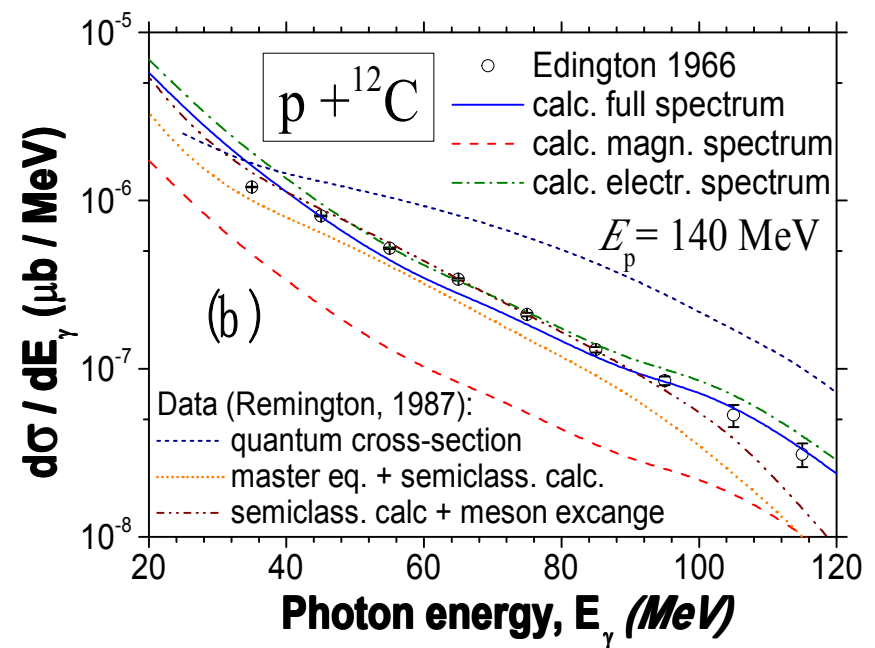
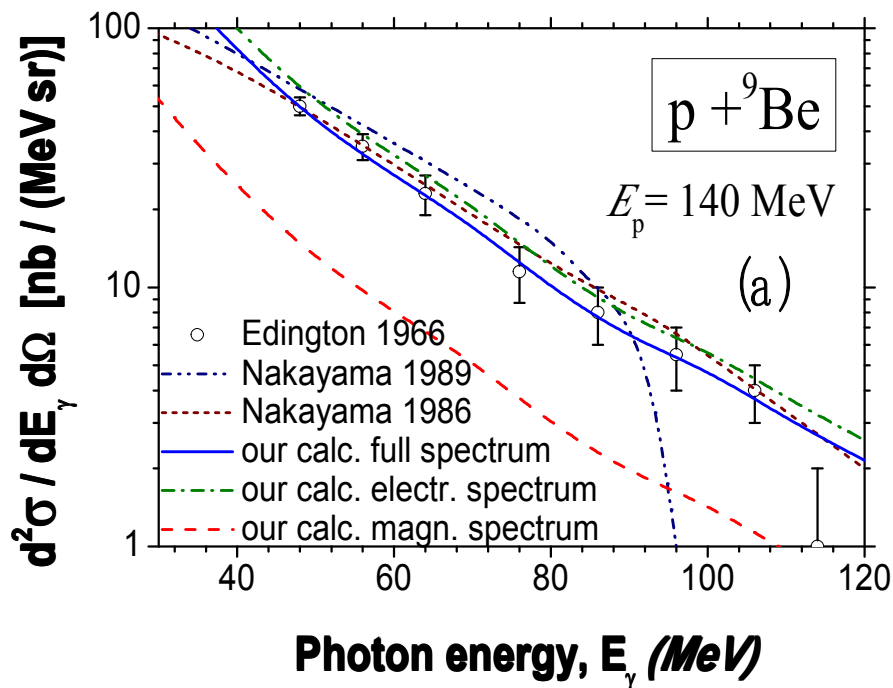
Aim: To study internal structure (many-nucleon interactions, quarks).

Target:  ${}^9\text{Be}$ ,  ${}^{12}\text{C}$ ,  ${}^{208}\text{Pb}$ .

Incident proton energy  $T_{\text{lab}} = 140 \text{ MeV}$ ,

Interested energy of emitted photons  $E_{\text{ph}} = 100\text{-}150 \text{ MeV}$ ,

To measure spectra at angles 15, 30, 45, 75, 90, 135.



# Proposal: Photons in decays, fission

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- Alpha-decay of heavy nuclei:

Only 4 nuclei were measured:  $^{210}\text{Po}$ ,  $^{214}\text{Po}$ ,  $^{226}\text{Ra}$ ,  $^{244}\text{Cm}$ .

Spectra of photons - up to 1 MeV.

*There are no any angular information.*

- Cluster-decay, proton-decay of nuclei:

*Up to present day there are no any experimental information.*

We give predictions for some nuclei.

Spectra of photons - up to 1 MeV.

- Ternary fission of heavy nuclei:

*Up to present day there are no any experimental information.*

We give predictions for  $^{252}\text{Cf}$ .

Spectra of photons - up to 10 MeV.

- Synthesis of superheavy nuclei:

*Up to present day there are no detailed study.*

# International Symposium on Physics of Photons

*September 27-29, 2013, IMP, Lanzhou, China*

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ISPP13 will be dedicated to all aspects of physics of photons in all types of reactions from the smallest energies up to high. At the meeting recent progress in our understanding of photons in nuclear and particle processes will be presented. Symposium will cover such questions as strong and electroweak interactions, many-nucleons effects, dynamics of studied reactions, dissipative, quantum non-locality effects, vacuum polarization, light-by-light scattering, photon collider prospects, acceleration techniques, future accelerators, etc. Recent results on physics of photons in astrophysics, black holes, cosmology and other related topics will be discussed.

- In theory it could be interesting to analyze current progress in modern models, to see vital problems and open unresolved questions, to establish perspectives.
- In experimental study it could be interesting to summarize existed progress, to indicate vital problems and understand their reasons, to see ways for resolutions.

## **Topics:**

- 1) Photons in proton-proton (nucleon-nucleon) collisions
- 2) Photons in collisions of protons, neutrons and light fragments on nuclei
- 3) Photons in collisions between heavy ions and nuclei
- 4) Photons in decays of nuclei (proton-decay, alpha-decay, cluster decay)
- 5) Photons in fission, fusion and synthesis of superheavy
- 6) Quarks in bremsstrahlung processes in nuclear and particle collisions
- 7) Photons in cosmos

# International Symposium on Physics of Photons

*September 27-29, 2013, IMP, Lanzhou, China*

## **Advisory Committee Members (confirmed):**

- 1) S. R. Banerjee (Variable Energy Cyclotron Centre, Kolkata, India)
- 2) C. A. Bertulani (Texas A&M University-Commerce, Texas, USA)
- 3) G. Giardina (Istituto Nazionale di Fisica Nucleare, Catania & Messina Universita, Italy)
- 4) J. Kasagi (Laboratory of Nuclear Science, Tohoku University, Japan)
- 5) B. Kopeliovich (Universidad Tecnica Federico Santa María, Valparaíso, Chile)
- 6) G. Mandaglio (Istituto Nazionale di Fisica Nucleare, Catania & Messina Universita, Italy)
- 7) A. I. Milstein (Max-Planck Institute, Germany & Budker Inst. Nucl. Phys., Novosibirsk, Russia)
- 8) K. Nakayama (Georgia University, USA & Institut für Kernphysik, Germany)
- 9) V. S. Olkhovsky (Institute for Nuclear Research, Kiev, Ukraine)
- 10) I. Schmidt (Universidad Tecnica Federico Santa María, Valparaíso, Chile)
- 11) E. V. Tkalya (Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Russia)
- 12) V. G. Zelevinsky (National Superconducting Cyclotron Laboratory, Michigan State University, USA)

## **Homepage:**

<http://photon2013.csp.escience.cn>

## **Organizing Committee:**

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Sergei Maydanyuk (Co-Chair, KINR, Ukraine):

[maidan@kinr.kiev.ua](mailto:maidan@kinr.kiev.ua)

Ju-Jun Xie (scientific secretary, IMP, China):

[xiejujun@impcas.ac.cn](mailto:xiejujun@impcas.ac.cn)

Li-Ping Zou (secretary, IMP, China):

[zoulp@impcas.ac.cn](mailto:zoulp@impcas.ac.cn)

## **Checklist of Dates:**

- 15 of May – Conference homepage is launched
- 15 of May – Conference registration opens
- 15 of May – Submission of abstracts opens
- 25 of August – Deadline for registration conference
- 25 of August – Deadline for submission abstract
- 1 of September – Notification of abstracts acceptance
- 27-29 of September – ISPP 13 Conference
- 1 of November – Deadline for submission papers

**Proceedings:** All peer-reviewed accepted papers will be published in *Journal of Physics: Conference Series (IOP Conference Series)*. Publication of select papers will be considered in other journals (Special Issues).

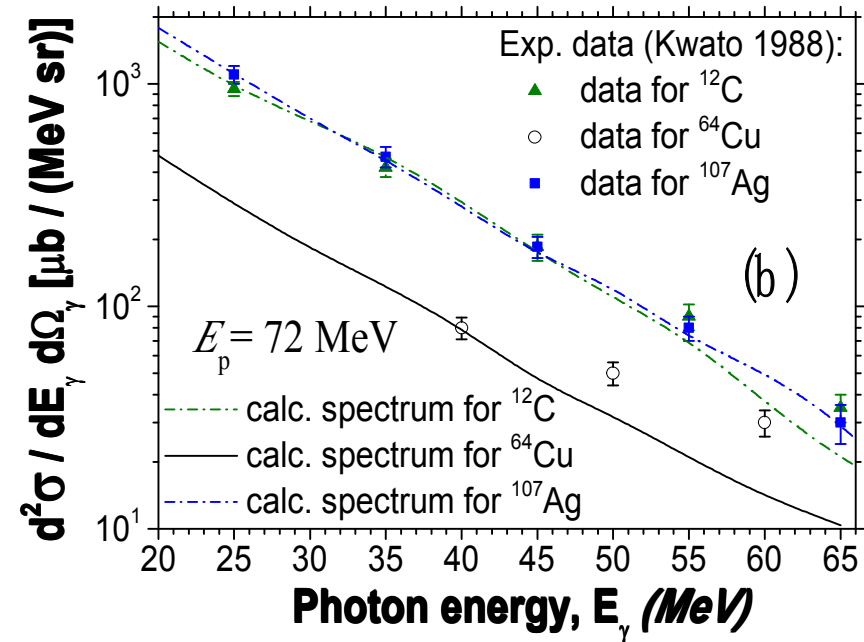
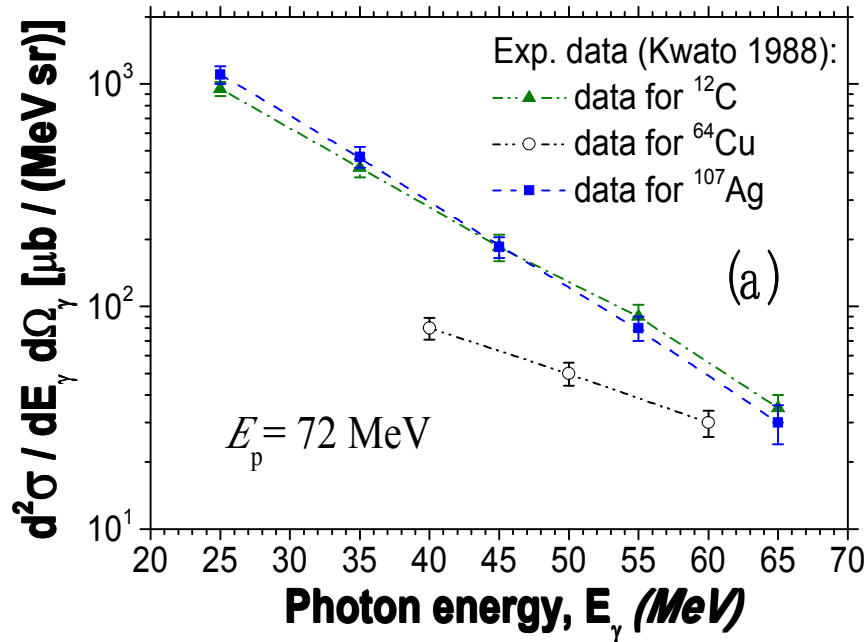


# Application of our model:

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1. Photons emitted in alpha-decay of nuclei
2. Photons emitted in decay of deformed nuclei
3. Photons emitted in spontaneous fission of nuclei
4. Photons emitted in ternary fission of nuclei
5. Photons emitted in interactions between protons and nuclei (proton decay of nucleus, scattering of proton off nuclei)

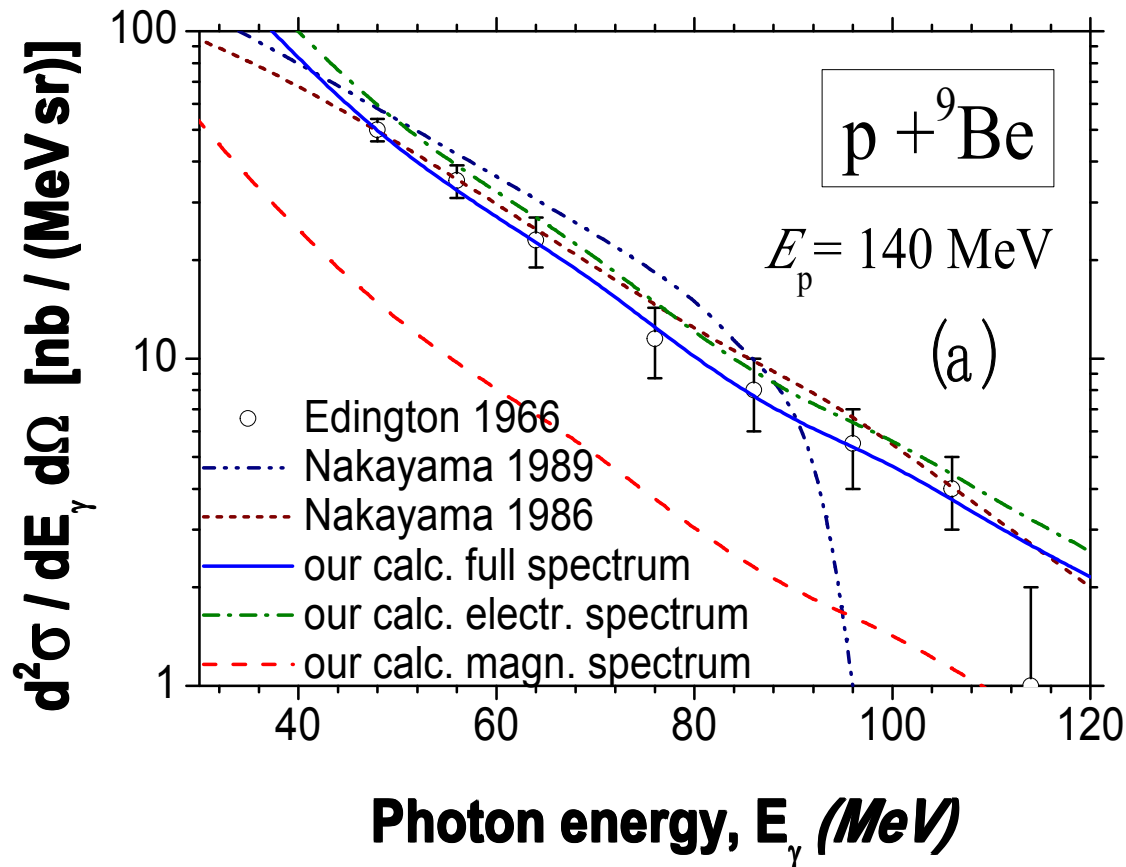
# Bremsstrahlung in proton nucleus collisions: incident energy $T_{\text{lab}} = 72 \text{ MeV}$



- (a) One can see that data for  $^{64}\text{Cu}$  are located lower than the data for  $^{12}\text{C}$  and  $^{107}\text{Ag}$ . At the same time, the data for  $^{64}\text{Cu}$  are decreased more slowly with increasing of the photon energy than the data for  $^{12}\text{C}$  and  $^{107}\text{Ag}$ .
- (b) The comparison between experimental data reinforced by calculations of the full cross-sections: inclusion of the calculated curves, describing general tendency of the spectra, only reinforces difference in behavior between experimental data, indicated on the previous figure (a).

[1] Kwato 1988: M.Kwato Njock et al., Phys. Lett. B207, 269 (1988).

# Bremsstrahlung in proton nucleus collisions: incident energy $T_{\text{lab}} = 140 \text{ MeV}$

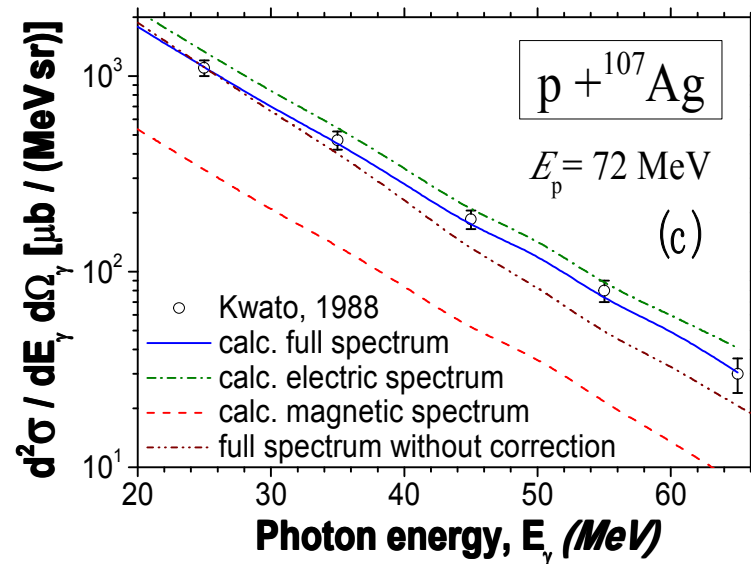
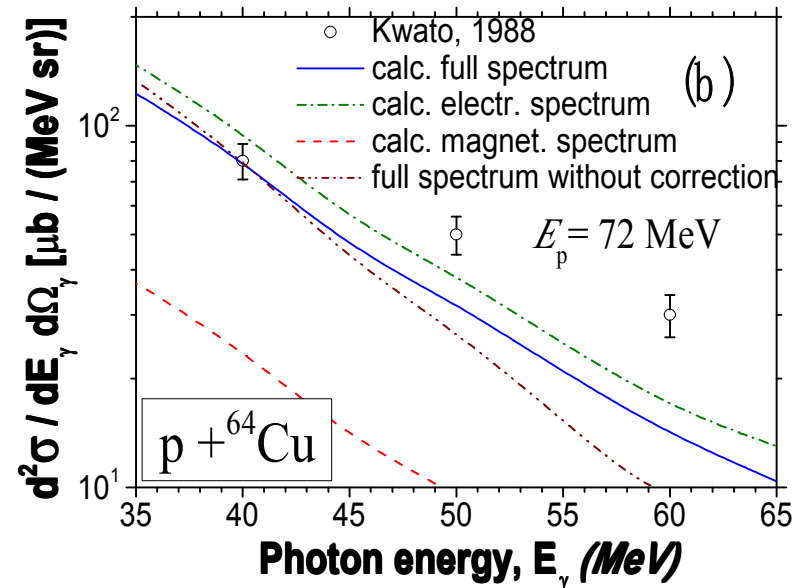
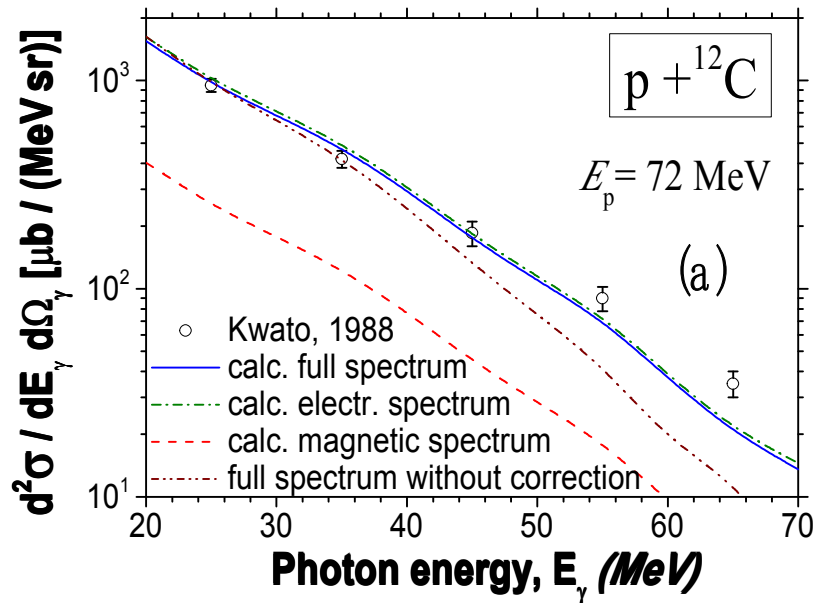


Comparison for  $p + {}^9\text{Be}$  between the calculations by our model [1], calculations from [2], calculations from [3] and experimental data [4]

- [1] Maydanyuk, Phys. Rev. **C86**, 014618 (2012): blue solid line is for full spectrum, green dash-dotted line for electric contribution, red dashed line for magnetic contribution
- [2] Nakayama 1986: K.Nakayama, G.Bertsch, Phys. Rev. **C34**, 2190 (1986).
- [3] Nakayama 1989: K.Nakayama, Phys. Rev. **C39**, 1475-1487 (1989).
- [4] Edington 1966: J.Edington, B.Rose, Nucl. Phys. **89**, 523 (1966).



# Bremsstrahlung in proton nucleus collisions: incident energy $T_{\text{lab}} = 72 \text{ MeV}$



[1] Maydanyuk, Phys. Rev. **C86**, 014618 (2012):  
wine dash double-dotted line is for full cross-section, blue solid line for corrected cross-section with division on  $k_f$ , green dash-dotted line for electric contribution, red dashed line for magnetic contribution.

[2] Kwato 1988: M.Kwato Njock et al., Phys. Lett. **B207**, 269 (1988).

# Our model

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## **Sketch:**

1. Two nucleons interactions are based on Dirac equation
2. Nuclear system – full set of interacting nucleons
3. Surface of nuclear system is changed dynamically
4. Potential of interaction between fragments (protons, nuclei) – tested by experimental data
5. Wave functions, boundary conditions
6. Bremsstrahlung probabilities
7. Calculations of spectra: for light and heavy masses of fragment, from zero up to high energies of photons emitted

# Free electron and Dirac equation

---

Symmetry between time and space:  $t \leftrightarrow r$

$$i \frac{\partial \Psi}{\partial t} = (-i\hbar \boldsymbol{\alpha} \frac{\partial}{\partial \mathbf{r}} + \beta m) \Psi = (\boldsymbol{\alpha} \mathbf{p} + \beta m) \Psi$$

Dirac equation.

Generalization on many-nucleons system:  $p \rightarrow P,$

$$i \frac{\partial \Psi}{\partial t} = (\boldsymbol{\alpha} \mathbf{P} + \beta m) \Psi = (\boldsymbol{\alpha} \sum_{i=1}^{A+1} \mathbf{p}_i + \beta m) \Psi$$

$\mathbf{P}$  – full momentum of many-nucleons system.

# Transformation of 4-component Dirac equation to 2-component dif. equation Model

Let's rewrite Dirac equation in the form:

$$i\frac{\partial\psi}{\partial t} = \left\{ c\boldsymbol{\alpha}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right) + \beta mc^2 + eA_0 \right\} \psi, \quad \mathbf{p} = -i\nabla. \quad \Psi = \psi \exp(imc^2 t) = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}.$$

If energy of proton is similar to its mass, i.e.  $|E-m| \ll m$ , then one spinor is essentially smaller than another.

Expansion of Dirac equation by powers  $1/c$ :  $i\frac{\partial\varphi}{\partial t} = \hat{H}\varphi,$

The first correction  
(Pauli equation)

$$\hat{H} = \frac{1}{2m} \left( \mathbf{p} - \frac{e}{c} \mathbf{A} \right)^2 + eA_0 - \mu \mathbf{H}, \quad \mu = \frac{e}{2mc} \boldsymbol{\sigma},$$

The second correction

$$\hat{H} = \frac{\mathbf{p}^2}{2m} + eA_0 - \frac{\mathbf{p}^4}{8m^3 c^2} - \frac{e}{4m^2 c^2} \boldsymbol{\sigma} \cdot [\mathbf{E} \times \mathbf{p}] + \frac{e}{8m^2 c^2} \Delta A_0.$$

$-\mathbf{p}^4 / 8m^3 c^2$  - includes dependence of mass on velocity

$-e/4m^2 c^2 \cdot \boldsymbol{\sigma} \cdot [\mathbf{E} \times \mathbf{p}]$  - energy of interaction of moving magnetic dipole with electric field

$e/8m^2 c^2 \cdot \Delta A_0$  - space distribution of electromagnetic charge

# Operator of emission

We generalize Pauli equation for  $A+1$  nucleons of the proton-nucleus system in laboratory frame:

$$\hat{H} \varphi = E \varphi, \quad \hat{H} = \sum_{i=1}^{A+1} \left\{ \frac{1}{2m_i} \left( \mathbf{p}_i - \frac{z_i e}{c} \mathbf{A}_i \right)^2 + z_i e A_{i,0} - \frac{z_i e \hbar}{2m_i c} \boldsymbol{\sigma} \cdot \text{rot}_i \mathbf{A}_i \right\} + V(\mathbf{r}_1 \dots \mathbf{r}_{A+1}).$$

$\Psi = (\chi, \varphi)$  is bispinor wave function of the proton-nucleus system,  $m_i$  and  $z_i$  are mass and charge of nucleon with number  $i$ ,  $\mathbf{A}_i$  is component of potential of the electromagnetic field formed by this nucleon (describing possible bremsstrahlung emission of photon caused by this nucleon),  $\boldsymbol{\sigma}$  are Pauli matrixes,  $A$  is mass number of nucleus,  $V(\mathbf{r}_1 \dots \mathbf{r}_{A+1})$  is potential of (nuclear, Coulomb) interactions between all nucleons.

In center-of-mass frame:

$$\mathbf{R}_A = \frac{1}{M} \sum_{j=1}^A m_j \mathbf{r}_{Aj}, \quad \mathbf{R} = \frac{M \mathbf{R}_A + m_p \mathbf{r}_p}{M + m_p}, \quad \mathbf{r} = \mathbf{r}_p - \mathbf{R}_A, \quad \boldsymbol{\rho}_j = \mathbf{r}_j - \mathbf{R}_A.$$

$$\hat{H} \psi = E \psi, \quad \hat{H} = \left\{ \frac{1}{2m} \left( \mathbf{p} - \frac{z e}{c} \mathbf{A} \right)^2 + z e A_0 - \frac{z e \hbar}{2m c} \boldsymbol{\sigma} \cdot \text{rot} \mathbf{A} \right\} + V(\mathbf{r}) + H_{\text{corr}}(\boldsymbol{\rho}_1 \dots \boldsymbol{\rho}_A).$$

Operator of perturbation:

$$\hat{W} = Z_{\text{eff}} \frac{e}{m c} \sqrt{\frac{2\pi \hbar c^2}{w_{\text{ph}}}} \sum_{\alpha=1,2} e^{-i\mathbf{k} \cdot \mathbf{r}} \left( i \mathbf{e}^{(\alpha)} \nabla - \frac{1}{2} \boldsymbol{\sigma} \cdot [\nabla \times \mathbf{e}^{(\alpha)}] + i \frac{1}{2} \boldsymbol{\sigma} \cdot [\mathbf{k} \times \mathbf{e}^{(\alpha)}] \right)$$

# Joint project

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## *Scheme of joint group*

Kiev's group [1]:  
S.P.Maydanyuk,  
V.S.Olkhovsky

- theory,
- calculations,
- predictions

Italian group [2]:  
G.Giardina, G.Mandaglio,  
PhD students

analysis of  
experimental data

Russian group [3]:  
N.Eremin, S.Paskhalov,  
D.Smirnov, S.Markochev etc.

experimental facilities  
( $\alpha$ -decay, fission),  
measurements

Chinese group:  
experimental facilities – that you have,  
measurements – everything.

- We have well working theory.
- We need in experimental group, and any experimental news.

[1] Institute for Nuclear Research, Kiev, National Academy of Science of Ukraine

[2] Istituto Nazionale di Fisica Nucleare, Sezione di Catania, and  
Dipartimento di Fisica dell'Universita di Messina, Italy

[3] Skobeltsin Institute of Nuclear Physics, Lomonosov Moscow State University, Russia.

# Ternary fission: Geometry of fissioning nuclear system

**Model**

1. We define geometry of nucleus separating on 3 fragments:

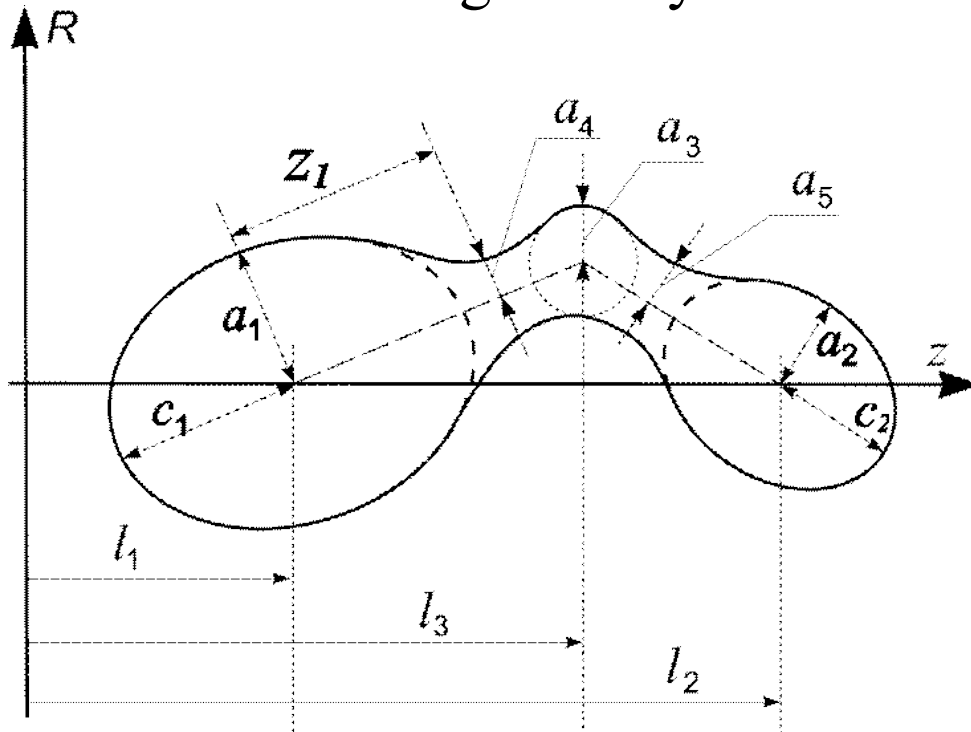


Fig.1: Geometry of nuclear system

2. We calculate  $\alpha$ -nucleus potential by standard folding procedure:

$$V_N(\mathbf{r}, \mathbf{R}_{12}) = V_{N,nucl}(\mathbf{r}, \mathbf{R}_{12}) - E_{N,frag},$$

$$V_C(\mathbf{r}, \mathbf{R}_{12}) = V_{C,nucl}(\mathbf{r}, \mathbf{R}_{12}) - E_{C,frag},$$

$$V_{N,nucl}(\mathbf{r}, \mathbf{R}_{12}) = -\lambda_N \int_{V(\mathbf{R}_{12})} \frac{d\mathbf{r}_1^3}{1 + \exp |\mathbf{r} - \mathbf{r}_1|}, \quad E_{N,frag} = -\lambda_N \int_V \frac{d\mathbf{r}^3}{1 + \exp |\mathbf{r}|},$$

$$V_{C,nucl}(\mathbf{r}, \mathbf{R}_{12}) = \lambda_C \int_{V(\mathbf{R}_{12}), \mathbf{r} \neq \mathbf{r}_1} \frac{d\mathbf{r}_1^3}{|\mathbf{r} - \mathbf{r}_1|}, \quad E_{C,frag} = \lambda_C \int_{V, \mathbf{r} \neq 0} \frac{d\mathbf{r}^3}{|\mathbf{r}|}.$$

# Motivations

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- Which aspect in interactions between proton and nucleus is more important?
  - Interaction between two nucleons is putted into basis of relativistic models of collisions, with application of formalism of Feynman's diagrams.
  - Consideration of nucleus as medium allows to study space distribution of all nucleons. Such a way takes into account non-locality of quantum mechanics, one of its fundamental aspects.
- Collective effects :
  - Models with nucleon-nucleon interaction should be the most accurate, if the collective effects caused by interactions between nucleons were small. However, we know that this is not so at low energies.
  - Many-nucleons interactions do not disappear at increasing of energy of nucleons. As we find, many-nucleons effects should arise at increasing of energy of photons emitted during nucleons motion.
  - There are indications on essential influence of many-nucleons interactions on the process of emission (two-nucleons approaches do not give positive explanation of nature of hard photons).



# Motivations

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- Properties of the bremsstrahlung accompanying scattering of protons off nuclei have been studied enough well [1,2]. As emitter of photons, both nucleus as medium, and different nucleons in it are considered. The process of emission is studied as result of deceleration of motion of nucleons in the averaged field of nucleus or in consequence of nucleon-nucleon collisions.
- Properties of the nuclear bremsstrahlung emission accompanying nucleon-nucleus and nucleus-nucleus collisions (especially, in region of intermediate energies up to 150 MeV / nucleon) have been studied worst of all [3].
- This forms our interest in use of the optical and folding potentials for investigations of the bremsstrahlung emission, which accompanies interactions protons with nuclei. It could be interesting to obtain the model, which allows to describe the spectra in energy region from minimal up to intermediate. Possibility to take quantum non-local properties into account in description of such interactions reinforces our interest in such potential approach.
- Microscopic models provide a powerful formalism for study of many-nucleons interactions, where wave functions were obtained from a single-configuration resonating group calculations. But, magnetic emission was not included into models, applied for description of the bremsstrahlung emission during scattering of protons on alpha-particles and light nuclei.

[1] V.A.Pluyko, V.A.Poyarkov, Phys. El. Part. At. Nucl. **18**, 374-418 (1987).

[2] V.V. Kamanin et al., Phys. El. Part. At. Nucl. 20, 743-829 (1989).

[3] I.V.Kopitin et al., Yad. Fiz. **60**, 869-879 (1997).

# Motivations

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- The magnetic emission is connected with magnetic momentum and spin of the fragment, interacting with nucleus. Attempt to take such aspects into account leads to matrix form of equations of interactions and many-component wave function of nuclear system [1]. However, the magnetic emission and spin formalism are included in relativistic models of collisions of nucleons between themselves and with nuclei at intermediate energies (based on Dirac equation).
- But, emphasis in literature (as [2,3]) was made on construction of relativistic description of interaction between two nucleons (in momentum representation).
- So, theory, combining spin formalism of interacting fragments of nuclear system (with inclusion of magnetic momentum) and potential approach for description of interaction between themselves, should be constructed.

[1] A.I.Ahiezer, V.B.Berestetskii, Kvantovaya Elektrodinamika (Nauka, Moskva, 1981).

[2] K.Nakayama, PR **C39**, 1475 (1989); V.Herrmann et al., PR **C43**, 394 (1991).

[3] M.K.Liou et al., PR **C35**, 651 (1987); Y.Li et al., PR **C72**, 024005 (2005); R.G.E.Timmermans et al., PR **C73**, 034006 (2006); Yi Li et al., PR **C84**, 034007 (2011).

# Summary (1)

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The new formalism for the bremsstrahlung emission accompanying proton decay and collisions of protons off nuclei in the energy region from the lowest up to intermediate, has been developed. This model includes spin formalism, potential approach for description of interaction between protons and nuclei, operator of emission includes magnetic emission.

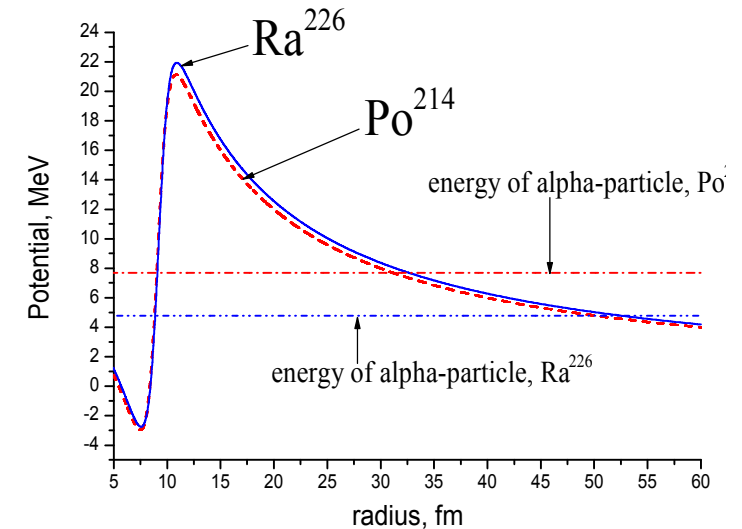
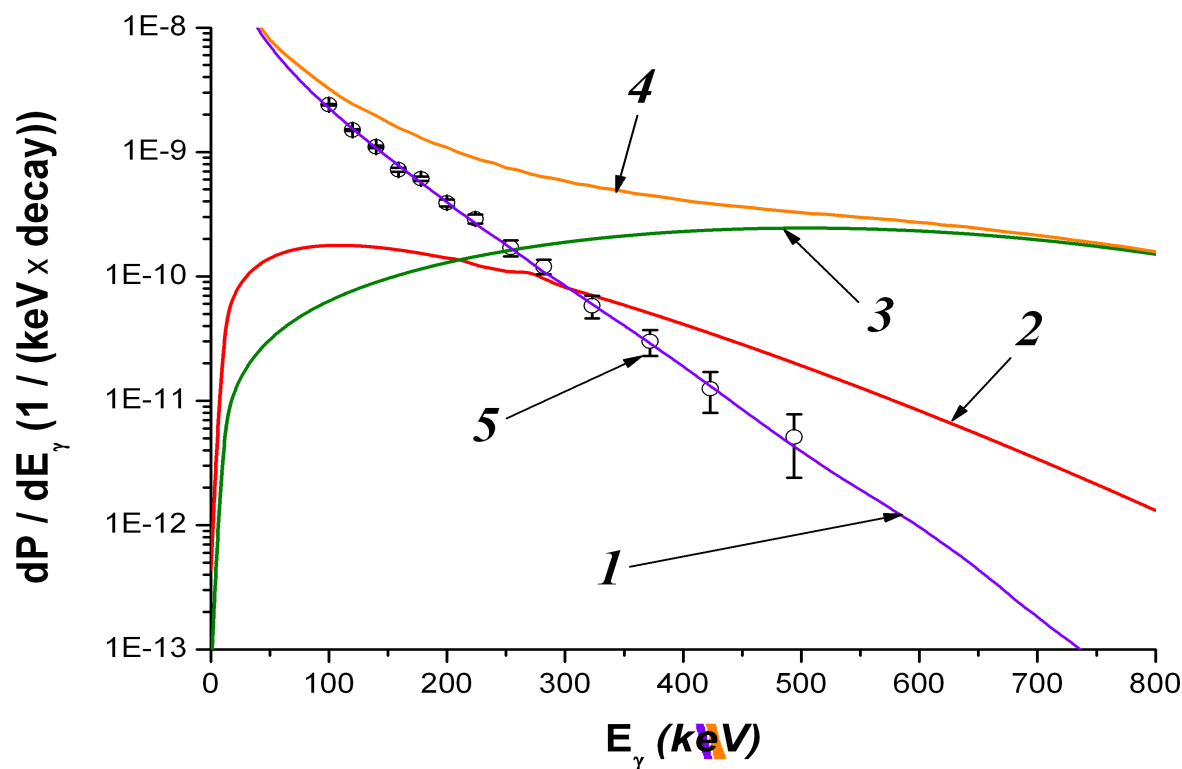
- In the problem of the bremsstrahlung during the proton decay in the first time a role of the magnetic emission is studied using such a model. For such investigations the  $^{146}\text{Tm}$  nucleus is chosen.
- Inside energy region up to 300 keV the magnetic emission gives contribution about 28 percents. However, the magnetic emission suppresses full emission probability ( $P_{\text{el}} / P_{\text{full}} = 1.14$ ). This effect of suppressing of the total emission can be explained by a presence of not small destructive interference between the electric and magnetic components inside whole studied energy region.
- ✓ With increasing of the angle between directions of the outgoing proton and emitted photon the electric and magnetic components increase proportionally, but ratio between them is not changed. So, there is no some angular value, where the magnetic emission increases essentially relatively electric one.

## Summary (2)

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- ✓ The magnetic component  $dP_{\text{mag},1}(r)$  is dependent on distance  $r$  between centers-of-masses of the proton and daughter nucleus similarly as the electric component  $dP_{\text{mag},1}(r)$  (ratio between such two components is not changed inside region from 5 fm up to 250 fm). In external region both components oscillate (having maxima and minima at similar space locations), while in the tunneling region they have monotonous shapes with one possible well. The emission from the internal region up to the barrier is the smallest, from external region - the strongest.
- ✓ With increasing of the angle between directions of the outgoing proton and emitted photon the electric and magnetic components increase proportionally, but ratio between them is not changed. So, there is no some angular value, where the magnetic emission increases essentially relatively electric one.
- ✓ The magnetic component  $dP_{\text{mag},1}(r)$  is dependent on distance  $r$  between centers-of-masses of the proton and daughter nucleus similarly as the electric component  $dP_{\text{mag},1}(r)$ . In the external region both components oscillate (having maxima and minima at similar space locations), while in the tunneling region they have monotonous shapes with one possible well. In general, the magnetic emission suppresses the full emission inside whole space region. The emission from the internal region up to the barrier is the smallest, and from the external region - the strongest.

# Emission from different regions for $^{210}\text{Po}$



**Bremsstrahlung during  $\alpha$ -decay:** (1) - total calc. spectrum; (2) – calc. curve of emission from tunneling region with boundaries  $R_{\text{tun},1}$  ( $E = 5,4$  MeV) and  $R_{\text{tun},2}$  ( $E = 5,4$  MeV); (3) - curve of emission from mixed region with boundaries  $R_{\text{tun},2}$  ( $E = 5,4$  MeV) and  $R_{\text{tun},2}$  ( $E = 4,4$  MeV); (4) - curve of emission from external region with boundaries  $R_{\text{tun},2}$  ( $E = 4,4$  MeV) and  $R_{\text{max}}$ ; (5) – exp. data in [1].

[1] J.Boie et al., Phys. Rev. Lett. **99**, 022505 (2007).