Chinese Physics C Vol. 50, No. 4 (2026) 044103

Editors' Suggestion

Spontaneous fission half-lives for heavy and super-heavy nuclei from phenomenological models*

Yi Xie (谢意)[1] Ning Wang (王宁)[1,2†] Zhongzhou Ren (任中洲)[3]

[1]Department of Physics, Guangxi Normal University, Guilin 541004, China
[2]Guangxi Key Laboratory of Nuclear Physics and Technology, Guilin 541004, China
[3]School of Physics Science and Engineering, Tongji University, Shanghai 200092, China

Abstract: A phenomenological model is proposed for a systematic description of the spontaneous fission (SF) half-lives $T_{\rm SF}$ of heavy and super-heavy nuclei. Based on the effective tunneling barrier (ETB), the proposed approach reproduces the SF half-lives of 79 known nuclei with an average deviation of 0.8, which is 17% smaller than that of the linear correlation approach recently proposed in [N. S. Moiseev, N. V. Antonenko, and G. G. Adamian, Phys. Rev. C **112**, 034607 (2025)]. For super-heavy nuclei with $45 \leqslant N-Z \leqslant 61$, the predicted SF half-lives from these two different phenomenological models are in close agreement. The ETB calculations imply that the β-decay energy affects the SF half-lives of nuclei far from the β-stability line. For super-heavy nuclei around the magic number $N = 184$, the predicted $T_{\rm SF}$ of $^{304}120$ is considerably shorter than that of ^{298}Fl. With predicted values of approximately 10 ~ 160 ms for $T_{\rm SF}$, the unmeasured SHN $^{293}119$ could survive for a sufficiently long time to reach the focal-plane detector in detection systems such as the gas-filled recoil separator SHANS in Lanzhou.

Keywords: spontaneous fission, super-heavy nuclei, tunneling barrier, β-decay energy, shell gap

DOI: 10.1088/1674-1137/ae3119 **CSTR:** 32044.14.ChinesePhysicsC.50044103

I. INTRODUCTION

As crucial and sensitive physical inputs, fission barriers and spontaneous fission (SF) half-lives of nuclei are frequently used in studies on nuclear physics [1−5], reactor physics [6], and nuclear astrophysics [7, 8]. Therefore, studies on nuclear fission are of significant interest [9−15], and the SF process [16−24] has been the subject of extensive investigations for nearly eight decades. The accurate prediction of SF half-lives in heavy and super-heavy nuclei (SHN) is essential for synthesizing new elements and understanding nuclear structure. SHN are typically produced via heavy-ion fusion reactions, forming excited compound nuclei that decay via evaporation or fission. For the successful identification in detection systems such as the gas-filled recoil separator SHANS in Lanzhou [25, 26], the compound nucleus must survive for longer than ~ 1 μs to reach the focal-plane detector. Consequently, accurate predictions of SF half-lives are indispensable for guiding experiments aimed at synthesizing new SHN and characterizing their decay chains.

Compared with α-decay, where theoretical models achieve relatively higher reliability [27−30], SF half-life predictions exhibit significant uncertainties, particularly for odd-A and odd-odd nuclei, where deviations from experimental data can span up to five orders of magnitude [20−22]. This discrepancy results from the complex, multi-dimensional nature of fission dynamics, involving uncertainties in fragment mass/charge distributions, neutron emission, and energy release. While α-decay can be effectively modeled as quantum tunneling through a one-dimensional barrier, fission involves traversing along the complicated potential energy surface (expressed in terms of several deformation parameters, and it is influenced by nuclear shell effects and pairing correlations) from the ground state to the scission point [31]. To compute deformed mean-field configurations and collective inertias, some microscopic approaches such as the constrained Hartree-Fock-Bogoliubov (HFB) method together with the Gogny energy density functional [32] and multidimensional constrained covariant density functional theory [33] have been developed. As a crude approximation, the SF half-lives ($T_{\rm SF}$) can be linked to the static fission barrier height $B_{\rm f}$ [34, 35]. However, as highlighted by

Received 13 November 2025; Accepted 25 December 2025; Accepted manuscript online 26 December 2025

* This work was supported by Guangxi "Bagui Scholar" Teams for Innovation and Research Project, National Key R&D Program of China (2023YFA1606503) and National Natural Science Foundation of China (12265006, 12035001, U1867212)

† E-mail: wangning@gxnu.edu.cn

Heßberger, T_{SF} is not uniquely determined by B_f alone. For example, the SF half-life of ^{236}U is up to nine orders of magnitude longer than that of ^{246}Cm despite the fission barrier heights [11] of these two nuclei ($B_f \approx 6$ MeV) being similar. This suggests that additional dynamical factors govern the fission process.

Drawing parallels to α-decay theory, Xu *et al*. proposed the relative Coulomb barrier height between the two fission fragments in comparison with the Q-value in SF as the key quantity for SF tunneling, and the penetration probability P of SF is expressed as [18]

$$P = \exp[-2\pi(V_{\rm top} - Q_{\rm sf})/\hbar\omega_{\rm f}]. \quad (1)$$

Here, $V_{\rm top}$ is the height of the Coulomb barrier between fission fragments, and $Q_{\rm sf}$ is the SF Q-value in the fission process. With an empirical formula for describing the Coulomb barrier,

$$V_{\rm top} = a_1A + a_2Z^2 + a_3Z^4 + a_4(N-Z)^2, \quad (2)$$

the values of $\log_{10} T_{\rm SF}$ for known even-even nuclei can be reproduced reasonably well (with an average deviation of 0.98). In addition, very recently, Moiseev *et al*. observed a linear correlation between $\log_{10} T_{\rm SF}$ and the corresponding α-decay energy Q_α for even-even nuclei with the same neutron excess $N-Z$,

$$\log_{10} T_{\rm SF} = b_0 + b_1(N-Z) + b_2(N-Z)^2 + b_3 Q_\alpha, \quad (3)$$

with which a phenomenological approach is presented for predicting the SF half-lives of actinides and SHN. The calculated half-lives closely matched the experimental data for known nuclei with an average deviation of 1.0 order of magnitude.

As a competition between the nuclear force and Coulomb repulsion, SF is strongly influenced by the isospin effect and the microscopic structure effects (*e.g.*, shell and pairing effects). In Eqs. (2) and (3), the isospin effect is represented by the neutron excess $(N-Z)$ terms. The microscopic effects are partly considered through Q_α in Eq. (3). For SHN and nuclei far from the β-stability line, the uncertainties of these two phenomenological formulas are considerably large. For example, Fig. 1 shows the predicted $\log_{10} T_{\rm SF}$ for even-even nuclei with $Z = 92$ and 114 using Eqs. (2) and (3). We can observe that the prediction discrepancies between two models are considerably large for unknown nuclei. Therefore, the accuracy of SF half-live predictions should be improved for unstable heavy and superheavy nuclei.

In this study, two phenomenological models are used to systematically investigate the trend of the SF half-lives of heavy and super-heavy nuclei, considering that the microscopic calculations are time-consuming. In addition, we study the influence of the β-decay energy Q_β, α-decay energy Q_α, and shell gap Δ that contains information about the microscopic shell and pairing energies on the SF half-lives of nuclei far from the β-stability line.

II. THEORETICAL FRAMEWORKS

In this study, we first investigate the trend of the measured SF half-lives for relatively stable nuclei. In Fig. 2(a), we show the maximum value of $\log_{10} T_{\rm SF}$ measured thus far in a certain isotopic chain as a function of neutron number. Additionally, Fig. 2(b) presents the corresponding relative barrier height $U_0 = V_{\rm top} - Q_{\rm sf} + \Delta$ and effective tunneling barrier U. Here, $V_{\rm top}$ and $Q_{\rm sf}$ denote the height of the Coulomb barrier between two fission fragments and corresponding SF Q-value in symmetric fission, respectively. The Coulomb barrier is expressed as $V_{\rm top} = Z_1Z_2e^2/(R_C^{(1)} + R_C^{(2)} + d)$ for symmetric fission, with the charge number $Z_1 \simeq Z_2$ of the fission fragments. The

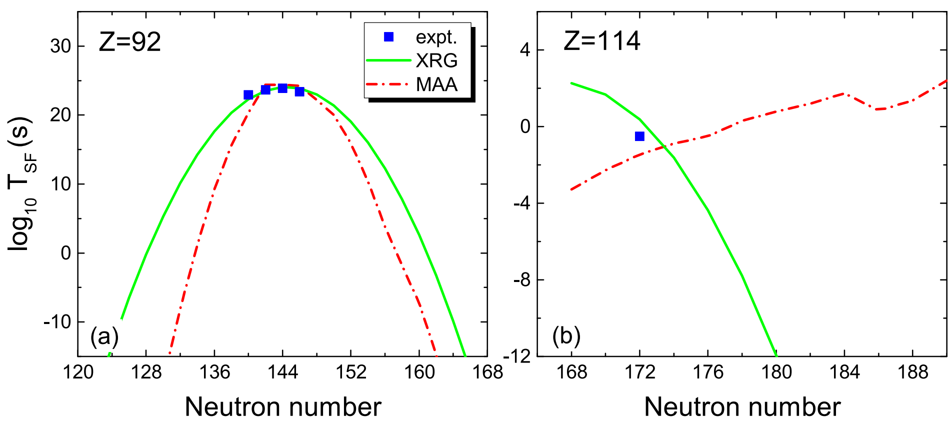

Fig. 1. (color online) Comparison of the predicted SF half-lives for even-even Uranium and Flerovium isotopes. The squares denote the experimental data taken from NUBASE2020 [36]. The solid and dot-dashed curves denote the results of XRG formula proposed in [18] and those of the MAA formula in [24], respectively.

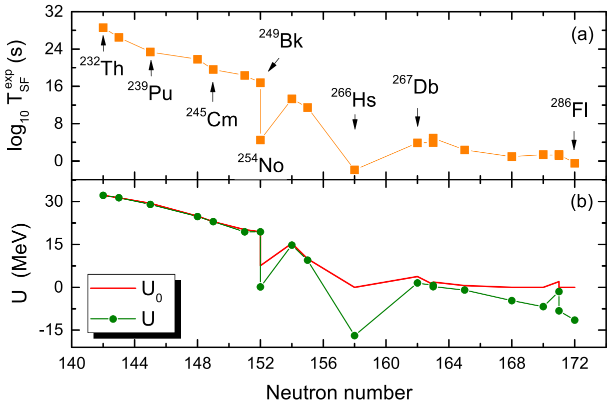

Fig. 2. (color online) (a) Maximum measured value of $\log_{10}T_{\rm SF}$ in each isotopic chain with mass numbers from $A=232$ to $A=286$. (b) Corresponding values of the effective tunneling barrier U. The red curve denotes the relative barrier height $U_0=V_{\rm top}-Q_{\rm sf}+\Delta$.

corresponding charge radius $R_C^{(1)}\simeq R_C^{(2)}$ of the fission fragments at their ground state are given by the WS charge radius formula [37]:

$$R_c=R_0\left[1+\frac{5}{8\pi}(\beta_2^2+\beta_4^2)\right],\tag{4}$$

with which the 1014 measured charge radii can be reproduced with an rms error of only 0.021 fm [38]. In Eq. (4), the nuclear charge radius R_0 at spherical shapes is given by

$$R_0=1.226A^{1/3}+2.86A^{-2/3}-1.09(I-I^2)+0.99\Delta E/A.\tag{5}$$

Here, nuclear quadrupole deformation β_2, hexadecapole deformation β_4, and shell correction ΔE are taken from the WS4 model [39]. $I=(N-Z)/A$ denotes the isospin asymmetry. In the calculations of the Coulomb barrier $V_{\rm top}$, we introduce a separation distance d between the fission fragments, considering that the fission fragments are in elongated shapes at the scission point. In addition, to consider the increase in the barrier height owing to the effect of unpaired nucleons [10] (which will be discussed later), we adopt different values for the separation distance. $d=2.12$ fm for even-even nuclei and $d=1.83$ fm for nuclei with unpaired nucleons. The difference of the separation distance d results in an approximately 2.1% change of the Coulomb barrier height for even-even actinides compared with that for the neighboring odd-A nuclei, with $R_C^{(1)}+R_C^{(2)}\approx 12$ fm. We note that the average inner fission barrier height [11] for odd-A actinides is higher than that of even-even actinides by approximately 2.5%, which indicates that adopting different separation distances is reasonable.

Δ denotes the shell gap in the parent nucleus, which is adopted to consider the influence of microscopic structure effects on the barrier and obtained from the difference of nuclear ground state energies [40]:

$$\Delta(N,Z)=E(N+2,Z)+E(N-2,Z)+E(N,Z+2)+E(N,Z-2)-4E(N,Z).\tag{6}$$

In addition to the barrier height, the barrier width can affect the fission probability according to WKB calculations. From the degree-of-freedom of elongation, the fission barriers of nuclei with spherical shapes can be thicker than those with prolate deformations. The shell gaps in doubly-magic nuclei (with spherical shapes) are generally larger than those in mid-shell nuclei (with prolate deformations). Therefore, to effectively consider the influence of barrier width, we add the shell gap Δ in the calculations of the relative Coulomb barrier height U_0.

The effective tunneling barrier (ETB) is defined as $U=U_0-Q_\beta/x$, in which the influence of isospin effects on the barrier height is considered for nuclei far from the β-stability line. Here, Q_β denotes the total β-decay energy for a nucleus, which is obtained by calculating the difference between the ground state energy of the parent nucleus and that of the most stable (lowest-energy) isobar with the same mass number. $x=\frac{E_c}{2E_s}$ denotes the fissionability parameter [9], with the Coulomb energy $E_c=a_cZ^2/A^{1/3}$ and surface energy $E_s=a_sA^{2/3}(1-\kappa I^2)$ of the nuclei, whose coefficients are taken from WS4 [39].

From Fig. 2, we observe that the trend of the effective tunneling barrier U agrees closely with that of $\log_{10}T_{\rm SF}$, including the abrupt falls of the half-lives for ^{254}No and ^{266}Hs, which implies that the effective tunneling barrier plays a role in the SF half-lives. Comparing U with U_0, we observe that the abrupt falls of the half-lives for neutron-deficient ^{254}No and ^{266}Hs could be due to the β-decay energy Q_β, which provides an additional energy to overcome the barrier as in the β-delayed fission process. Considering the possible correlations between ETB and $\log_{10}T_{\rm SF}$, the SF half-lives for heavy and super-heavy nuclei are expressed as

$$\log_{10}T_{\rm SF}(s)=c_1U+c_2U^2+\Delta_{\rm res}.\tag{7}$$

The term $\Delta_{\rm res}$ is introduced to consider the influence of the pairing effects and residual correlations:

$$\Delta_{\rm res}=\begin{cases}\Delta+\sin(c_3Q_\alpha) & : N \text{ and } Z \text{ even}\\ \frac{3}{2}\Delta+\sin(c_3Q_\alpha+\frac{1}{2}\Delta) & : A \text{ odd}\\ 2\Delta+\sin(c_3Q_\alpha+\frac{1}{2}\Delta) & : N \text{ and } Z \text{ odd.}\end{cases}\tag{8}$$

where the Q_α term is an empirical correction to consider the correlation between $\log_{10} T_{\rm SF}$ and Q_α [24], which will be discussed later. By fitting the measured SF half-lives for 79 known nuclei with certain branching ratios in NUBASE2020 [36], we obtain the optimal model parameters $c_1 = 0.466$, $c_2 = 0.0088$, and $c_3 = 1.98$.

For nuclei with odd proton and/or odd neutron numbers, the change in the energy of the fissioning state along the fission path plays a decisive role. Owing to spin and parity conservation at crossing points of Nilsson levels, the unpaired nucleon generally cannot change the level, unlike nucleon pairs in even-even nuclei, which leads to an effective increase in the fission barrier [21]. Therefore, the half-lives of odd-mass nuclei are systematically longer than their neighboring even-even nuclei owing to the unpaired nucleons [41]. In this work, different values of $\Delta_{\rm res}$ are adopted to consider the pairing effects.

For even-even parent nuclei, the unpaired nucleons of fission fragments could lead to a relatively higher barrier in symmetric fission due to the lower Q-value. To consider the influence of the unpaired nucleons, in the calculations of U_0, we take the mean value of the $Q_{\rm sf}$ among three cases: with completely symmetric fragments, with one more neutron for a fragment, and with one more proton for a fragment. As an example, the three channels ^{254}Fm $\rightarrow ^{127}$Sn+^{127}Sn, ^{254}Fm $\rightarrow ^{128}$Sn+^{126}Sn, and ^{254}Fm $\rightarrow ^{127}$Sb+^{127}In are considered in the calculations of U_0 for ^{254}Fm. We note that taking the mean value for $Q_{\rm sf}$ can evidently improve the model accuracy for even-even nuclei. In addition, in the calculations of U_0, we introduce a truncation, *i.e.*, $U_0 \geqslant 0$, to avoid a negative barrier height.

III. RESULTS AND DISCUSSIONS

In Fig. 3, we show the predicted spontaneous fission half-lives of even-even actinides as a function of neutron numbers. The curves denote the results calculated using Eq. (7). Open symbols denote the experimental data taken from NUBASE2020. From Fig. 3, we observe that the measured SF half-lives can be reproduced reasonably well. In addition, we note that the SF half-lives of neutron-deficient Cf and Fm isotopes are evidently shorter than those of the corresponding nuclei around the β-stability line, which also indicates that the β-decay energy Q_β may play a role in the decreasing behavior of SF half-lives for nuclei far from the β-stability line.

Figure 4 shows the deviations between the measured SF half-lives and model predictions. The circles and squares denote the results obtained with the proposed ETB approach and those of the MAA formula [24], respectively. For almost all nuclei, the deviations between the calculated half-lives and experimental data are within three orders of magnitude. Through Eq. (9), the average deviation between ETB predictions and experimental data

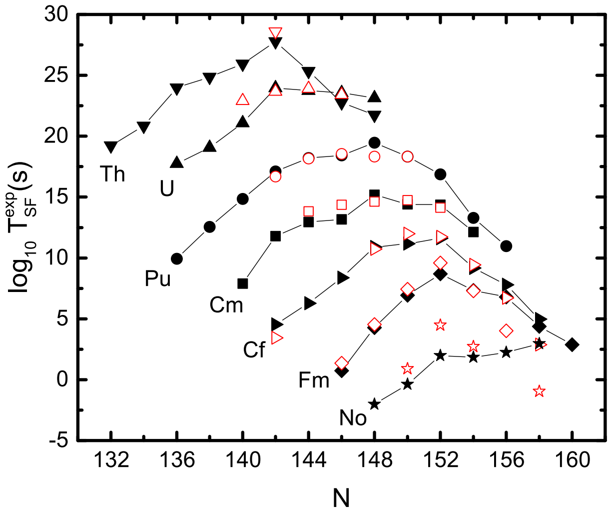

Fig. 3. (color online) Spontaneous fission half-lives of even-even actinides. The black curves denote the results calculated with Eq. (7), and the red open symbols denote the experimental data taken from NUBASE2020 [36].

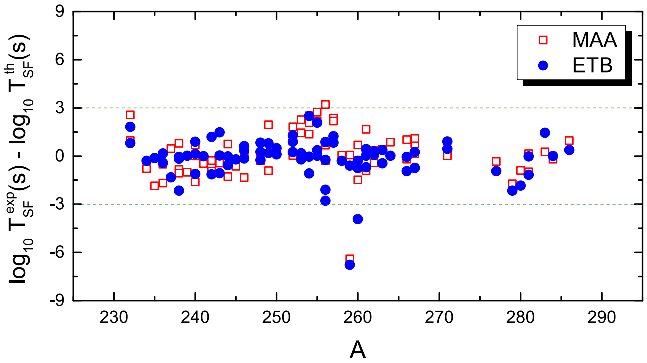

Fig. 4. (color online) Deviations of model predictions from the experimental half-lives. The circles and squares denote the results obtained with the proposed ETB approach and those of the MAA formula [24], respectively.

$$\langle\sigma\rangle = \frac{1}{n}\sum_{i=1}^{n}\left|\log_{10}(T_{\rm SF}^{{\rm th},i}/T_{\rm SF}^{{\rm exp},i})\right| \tag{9}$$

is only 0.800 for the 79 known nuclei. For ^{259}Fm, we note that its SF half-life is significantly over-predicted by both MAA and ETB calculations. In Table 1, we list the calculated average deviations $\langle\sigma\rangle$ with respect to the SF half-lives of 79 nuclei (including 42 even-even nuclei) using two phenomenological models (MAA and ETB). In the calculations, the α-decay energy Q_α are taken from WS4 plus radial basis function (WS4+RBF) predictions [39], with which the known α-decay energies of SHN can be reproduced with an rms error of 0.22 MeV [28]. For 42 even-even nuclei, the average deviation is $\langle\sigma\rangle = 0.783$ with the proposed ETB approach, which is smaller than that with the XRG formula [18] and MAA formula [24] by 28% and 13%, respectively. Note that, in [24], the SF

Table 1. Average deviation between model predictions and 79 experimental SF half-lives taken from NUBASE2020 [36]

model	even-even nuclei	all nuclei
MAA	0.900	0.959
ETB	0.783	0.800

half-lives of 111 nuclei (including 58 even-even nuclei) were used for analysis.

To investigate the residual correlations between $\log_{10} T_{\rm SF}$ and Q_α, Fig. 5 shows the deviations between the measured half-lives and the predictions using the ETB approach but neglecting the Q_α term in the calculations. A somewhat oscillatory behavior occurs in the deviations. In Eq. (8), we adopt a sine function to describe the correlations between $\log_{10} T_{\rm SF}$ and Q_α rather than a linear relationship, considering that many properties of atomic nuclei (such as binding energy and fission barrier) exhibit oscillatory behavior similar to a sine wave with changes in nucleon number, particularly when crossing shell closures. The shell correction energy itself represents an oscillation around the smooth values predicted by the liquid-drop model. The sine function captures the oscillatory shell corrections, similar to its use in modeling the residual shell effects in nuclear masses [42]. In addition, compared with the linear correlation, the sine function is a bounded function with a range of $[-1,1]$. This means that regardless of how large Q_α becomes, the contribution of the correlation term always has an upper and lower limit, indicating the existence of a saturation mechanism with which the unphysical and infinite contribution can be avoided. With the phenomenological sine functions to describe the oscillatory behavior, the average deviation $\langle\sigma\rangle$ is significantly reduced by approximately 20%.

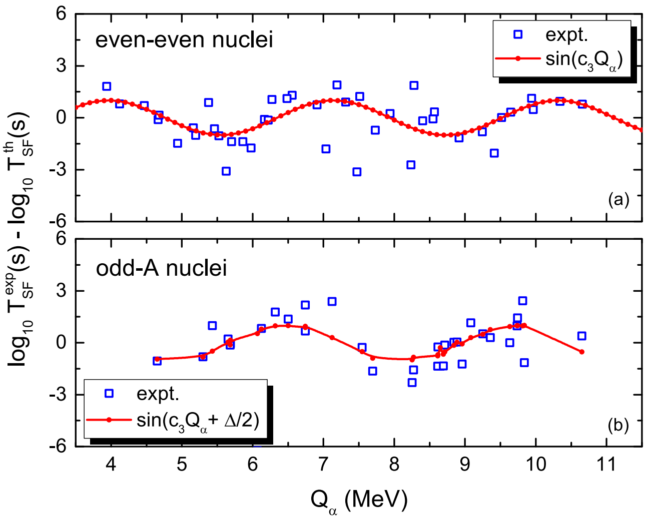

Fig. 5. (color online) Deviations between the measured half-lives and the predictions with Eq. (7) but neglecting the Q_α term. (a) for even-even nuclei and (b) for odd-A nuclei.

In Fig. 6, we show the predicted spontaneous fission half-lives for Am, Es, Db, and Mc isotopes. The squares denote the data taken from NUBASE2020. We observe that the SF half-lives can be reproduced well with both models for known nuclei. For Am and Es isotopes in the range of $40<N-Z<62$, the predicted results from the two models agree closely in general. For Am and Es isotopes far from the β-stability line, the predicted SF half-lives with the MAA formula are significantly smaller than those with Eq. (7). For some extremely neutron-deficient nuclei such as ^{214}U [4], we note that the predicted SF half-lives from the MAA formula are unphysical and catastrophically short (see Fig. 1). For the MAA model, the authors emphasized that extrapolation to nuclei well outside the range $40<N-Z<62$ should be treated with caution because the linear correlation used in the formulas is observed from the available experimental SF half-lives in the following ranges: $90\leqslant Z\leqslant 102$ and $41\leqslant N-Z\leqslant 60$, as well as $103\leqslant Z\leqslant 118$ and $45\leqslant N-Z\leqslant 61$. In the ETB calculations, we introduce a truncation $Q_\beta\leqslant a_{\rm sym}$ for nuclei around driplines, with the symmetry energy coefficient $a_{\rm sym}=c_{\rm sym}\left[1-\frac{\kappa}{A^{1/3}}+\xi\frac{2-|I|}{2+|I|A}\right]$ taken from WS4. $a_{\rm sym}$ represents the upper energy limit of neutron-proton asymmetry that a nuclear system can sustain. The truncation for Q_β ensures that the model's predictions, even under extreme conditions, do not deviate entirely from physical reality. For Db isotopes, the results from the two models are considerably close, and the peaks at $N=162$ and $N=178$ are due to the shell effects in the calculations of ETB. For Mc isotopes around the neutron magic number $N=184$, the predicted SF half-lives with the proposed method are significantly enhanced owing to the strong shell effects.

Figure 7 shows the predicted spontaneous fission half-lives for Sg, Ds, Fl, and Og isotopes. We note that the available experimental data from NUBASE2020 can be well reproduced by these two phenomenological models. Additionally, the odd-even staggering due to the pairing effects can be clearly observed from the results of both models. For Fl and Og isotopes, the predicted SF half-lives around $N=184$ with ETB are significantly larger than those with the MAA formula. In the region of $45\leqslant N-Z\leqslant 61$, the predicted results from these two models agree closely. From Fig. 7 and Fig. 8 in Ref. [40], we note that the uncertainties of the predicted shell gaps from different mass models are still large for SHN. The obtained shell gaps from WS4 are $\Delta=3.40$ MeV for ^{272}Ds, 5.58 MeV for ^{298}Fl, and 3.76 MeV for ^{304}120. The pronounced enhancement of the SF half-lives for nuclei around ^{298}Fl is due to the large shell gaps because $\log_{10} T_{\rm SF}$ is directly related to Δ according to Eq. (8).

In Fig. 8, we present the predicted SF half-lives for nuclei with $Z=119$ and $Z=120$. The trends of the predicted SF half-lives for neutron-rich nuclei from the two

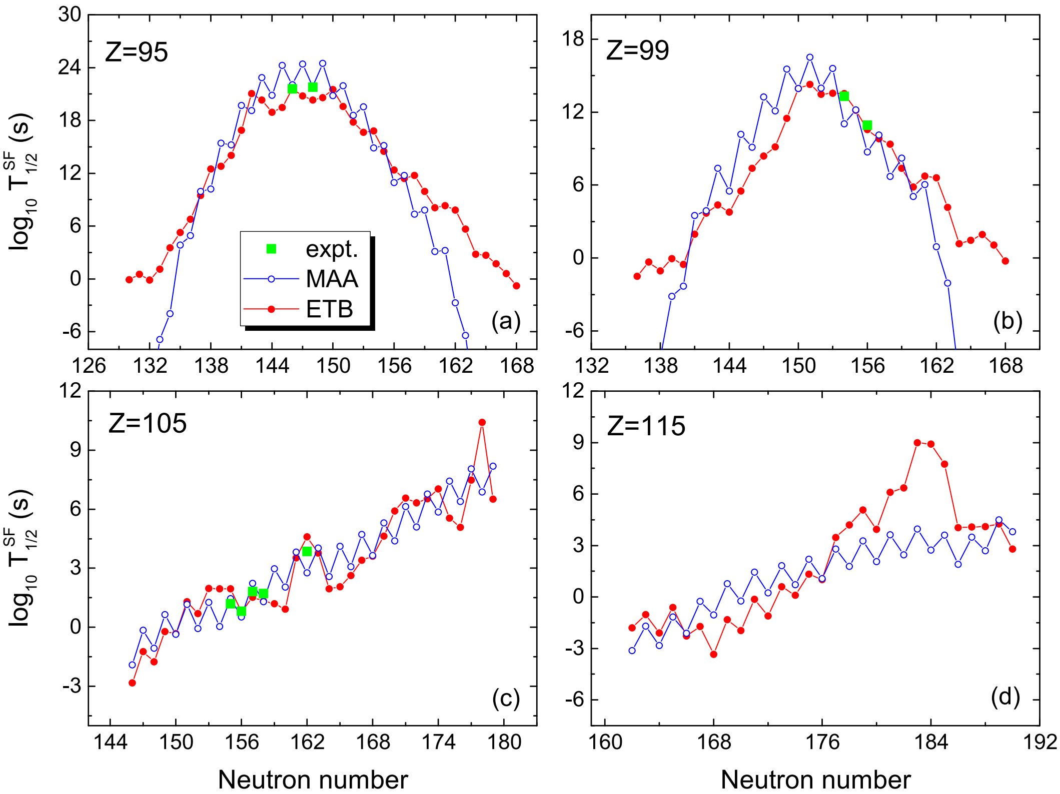

Fig. 6. (color online) SF half-lives for Am, Es, Db, and Mc isotopes. The red curves with solid circles denote the predictions with the ETB approach proposed in this work. The blue curves with open circles denote the results of MAA [24]. The squares denote the experimental data taken from NUBASE2020 [36].

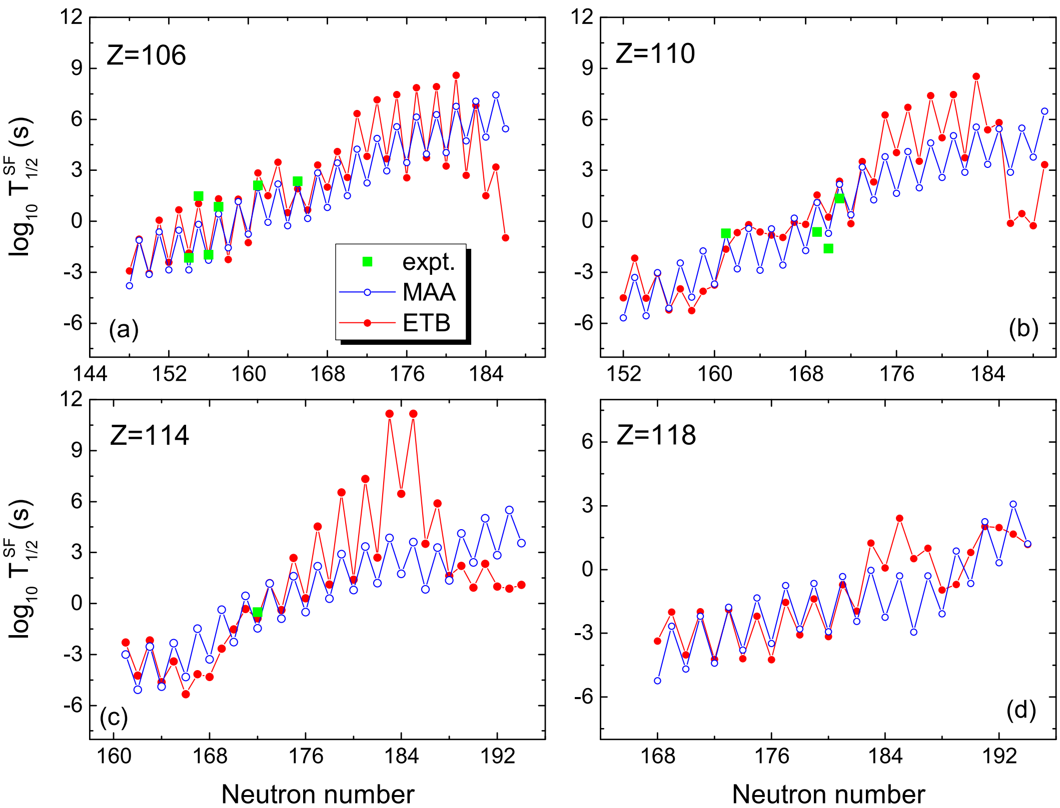

Fig. 7. (color online) Same as Fig. 6 but for Sg, Ds, Fl, and Og isotopes.

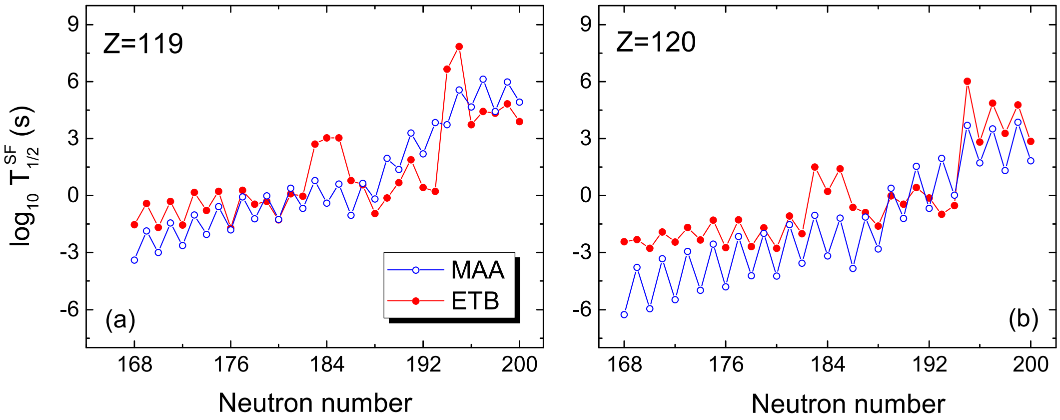

Fig. 8. (color online) Same as Fig. 6 but for nuclei with $Z = 119$ and $Z = 120$.

model are similar. At the neutron-deficient side, the results of the ETB approach proposed in this work are slightly higher than those of the MAA model. For the SHN $^{293}119_{174}$, the predicted value of $T_{\rm SF}$ is approximately 10 ~ 160 ms based on MAA and ETB calculations. Because the average deviations between data and model predictions from the two models are within one order of magnitude, the SHN $^{293}119$ that might be synthesized in the fusion reaction ^{54}Cr+^{243}Am after evaporating four neutrons can survive for much longer than ~1 μs to reach the focal-plane detector.

IV. SUMMARY

In this study, we have compared two phenomenological models for systematically describing the SF half-lives $T_{\rm SF}$ of heavy and superheavy nuclei. Based on the ETB, which considers the relative Coulomb barrier between fission fragments, the shell gap Δ, and the β-decay energy Q_β of the fissioning nuclei, the SF half-lives of 79 known nuclei can be reproduced with an average deviation of 0.8, which is smaller than those of two other phenomenological models: XRG and MAA formulas. With a value of $U = 28.8$ MeV, the height of the ETB for ^{236}U is higher than that of ^{246}Cm ($U = 19.5$ MeV) by 9.3 MeV, which explains why the SF half-life of ^{236}U is up to nine orders of magnitude longer than that of ^{246}Cm. The competition between Coulomb and isospin effects, represented by the β-decay energy Q_β, significantly impacts SF half-lives of nuclei far from the β-stability line. For superheavy nuclei with $45 \leqslant N-Z \leqslant 61$, the predicted SF half-lives from two different phenomenological models (ETB and MAA) are generally in good agreement. For nuclei around neutron magic numbers such as $N = 162$ and $N = 184$, the predicted SF half-lives from the ETB are larger than those from MAA because the shell gaps are directly involved in the ETB calculations. Both the proposed ETB approach and MAA formula predict a remarkably shorter SF half-life for the superheavy nucleus $^{304}120$ compared with that for ^{298}Fl. For nuclei with $Z = 119$ and $N = 174$, the predicted $T_{\rm SF}$ from the ETB and MAA approaches is approximately 10 ~ 160 ms, which is longer than the corresponding α-decay half-life [27] by two or three orders of magnitude. These results provide useful insights for future experiments aimed at synthesizing new superheavy elements and understanding their stability.

ACKNOWLEDGEMENTS

N. W. is grateful to Dr. Zhiyuan Zhang for valuable discussions. The predicted SF half-lives for 1311 nuclei are available on http://www.imqmd.com/decay/

References

[1] T. S. Fan, K. X. Jing, L. Phair *et al.*, Nucl. Phys. A **679**, 121 (2000)
[2] K. Zhao, Z. X. Li, N. Wang *et al.*, Phys. Rev. C **92**, 024613 (2015)
[3] Z. Y. Zhang *et al.*, Phys. Rev. Lett. **122**, 192503 (2019)
[4] Z. Y. Zhang *et al.*, Phys. Rev. Lett. **126**, 152502 (2021)
[5] S. Hofmann, S. Heinz, R. Mann *et al.*, Eur. Phys. J. A **52**, 116 (2016)
[6] C. Wagemans, M. Asghar, P. D'hondt *et al.*, Nucl. Phys. A **285**, 32 (1977)
[7] I. V. Panov *et al.*, Nucl. Phys. A **747**, 633 (2005)
[8] S. Goriely, Eur. Phys. J. A **51**, 22 (2015)
[9] S. Cohen and W. J. Swiatecki, Ann. Phys. (N.Y.) **22**, 406 (1963)
[10] S. Bjørnholm and J. E. Lynn, Rev. Mod. Phys. **52**, 725 (1980)
[11] R. Capote *et al.*, Nucl. Data Sheets **110**, 3107 (2009)
[12] A. N. Andreyev, K. Nishio, and K. H. Schmidt, Rep. Prog. Phys. **81**, 016301 (2018)
[13] Z. H. Liu, H. Q. Zhang, J. C. Xu *et al.*, Phys. Rev. C **54**, 761 (1996)
[14] Y. Qiang and J. C. Pei, Phys. Rev. C **104**, 054604 (2021)

[15] O. I. Davydovska, V. Yu. Denisov, and I. Yu. Sedykh, Phys. Rev. C **105**, 014620 (2022)
[16] W. J. Swiatecki, Phys. Rev. **100**, 937 (1955)
[17] Z. Ren and C. Xu, Nucl. Phys. A **759**, 64 (2005)
[18] C. Xu, Z. Ren, and Y. Guo, Phys. Rev. C **78**, 044329 (2008)
[19] K. P. Santhosh, R. K.Biju, and S. Sahadevan, Nucl. Phys. A **832**, 220 (2010)
[20] X. J. Bao, S. Q. Guo, H. F. Zhang *et al*., J. Phys. G: Nucl. Part. Phys. **42**, 085101 (2015)
[21] F. P. Heßberger, Eur. Phys. J. A **53**, 75 (2017)
[22] Z. Yuan *et al*., Eur. Phys. J. A **60**, 68 (2024)
[23] K. Pomorski *et al*., Eur. Phys. J. A **58**, 77 (2022)
[24] N. S. Moiseev, N. V. Antonenko, and G. G. Adamian, Phys. Rev. C **112**, 034607 (2025)
[25] H. B. Yang *et al*., Phys. Rev. Lett. **132**, 072502 (2024)
[26] H. B. Zhou, Z. G. Gan, N. Wang *et al*., Phys. Rev. C **103**, 044314 (2021)
[27] J. Tian, K. Ren, P. Ma *et al*., Phys. Rev. C **110**, 064313 (2024)
[28] Y. Z. Wang, S. J. Wang, Z. Y. Hou *et al*., Phys. Rev. C **92**, 064301 (2015)
[29] G. Royer, Nucl. Phys. A **848**, 279 (2010)
[30] X.-T. He *et al*., Phys. Rev. C **111**, 064312 (2025)
[31] R. Rodríguez-Guzmán and L. M. Robledo, Phys. Rev. C **89**, 054310 (2014)
[32] R. Rodríguez-Guzmán, Y. M. Humadi, and L. M. Robledo, Eur. Phys. J. A **56**, 43 (2020)
[33] B.-N. Lu, E.-G. Zhao, and S.-G. Zhou, Phys. Rev. C **85**, 011301 (2012)
[34] P. Möller *et al*., Phys. Rev. C **91**, 024310 (2015)
[35] N. Wang and M. Liu, Chin. Phys. C **48**, 094103 (2024)
[36] F. G. Kondev *et al*., Chin. Phys. C **45**, 030001 (2021)
[37] N. Wang and T. Li, Phys. Rev. C **88**, 011301 (2013)
[38] T. Li, Y. Luo, and N. Wang, Atom. Data Nucl. Data Tabl. **140**, 101440 (2021)
[39] N. Wang, M. Liu, X. Z. Wu *et al*., Phys. Lett. B **734**, 215 (2014)
[40] Q. Mo, M. Liu, and N. Wang, Phys. Rev. C **90**, 024320 (2014)
[41] R. Rodríguez-Guzmán and L. M. Robledo, Eur. Phys. J. A **52**, 348 (2016)
[42] N. Wang, Z. Liang, M. Liu *et al*., Phys. Rev. C **82**, 044304 (2010)