Collisions of very heavy nuclei $^{197}$Au + $^{197}$Au at the energy of 15A MeV has been studied with the improved quantum molecular dynamics model. The experimental mass distributions of ternary fission fragments for the system $^{197}$Au + $^{197}$Au are reproduced well. The direct and sequential ternary fission modes are studied by the time dependent snapshots of typical ternary events. The analysis of deviation from Viola systematics indicates the nonstatistical feature of the ternary fission in these reactions.

Keywords: ImQMD model; kinematic correlation; ternary fission; Viola systematics.

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1. Introduction

Ternary fission of heavy nuclei has been one of the interesting topics in contemporary nuclear physics in the last decades. Early investigations of the dynamics of ternary fission mainly concentrated on fission processes accompanied by light-particle emission, for example, $\alpha$-particle, Be and C etc.\textsuperscript{1–7} However, for superheavy systems with very massive charges generated by heavy ions reactions, there is very clear evidence for fission into three comparable fragments. The experiment on the ternary partitions of $^{197}$Au + $^{197}$Au at 15 A MeV was carried out by Skwira-Chalot et al.$^{8,9}$ in $4\pi$ geometry using the multidetector array CHIMERA at LNS Catania. The result of the experiment displayed the mass number distributions of fragments

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according to the mass $A_1$ (the heaviest), $A_2$ (the intermediate) and $A_3$ (the light-est), and the peak of mass number distribution of fragments $A_3$ was very close to 100. To understand this type of ternary fission, the two-center shell model was extended to the three-center case and the static behavior of ternary fission for the very heavy systems was studied within the framework of the liquid-drop model. Nevertheless, a knowledge of the potential energy surface alone is not sufficient to predict the dynamical behavior of the system.

It seems to be of great significance to investigate the possible ternary fission modes, since, up to now, little is known about the dynamics of ternary fission for such superheavy systems. In the macroscopic dynamical model, only the statistical fission can be described. In order to reveal the new feature of the ternary fission, for example, to reveal the nonstatistical feature of the fission, we have to employ the microscopic transport dynamical model (the improved quantum molecular dynamics (ImQMD) model). By using this microscopic dynamic model, the motions such as shape deformations, neck formation and rupture, nucleon transfer and so on are involved consistently. And there is no need to introduce the friction strengths and their form-factors, mass parameters, etc., which are known to have large uncertainty and have to be introduced in the macroscopic description. With this ImQMD model, the fusion dynamics at energies near and above the barrier has been extensively studied and can reproduce a series of experimental data.

2. Brief Introduction of the ImQMD Model

For self-containedness and reader’s convenience, let us first briefly introduce the ImQMD model. In the ImQMD model, the same as in the original QMD model, each nucleon is represented by a Gaussian wave packet,

$$\phi_i(r) = \frac{1}{(2\pi\sigma_i^2)^{3/4}} \exp \left[-\frac{(r - r_i)^2}{4\sigma_i^2} + \frac{i}{\hbar} r \cdot p_i \right], \quad (1)$$

where $r_i$, $p_i$, are the centers of $i$th wave packet in the coordinate and momentum space, respectively. $\sigma_r$ represents the spatial spread of the wave packet. The total $N$-body wave function is assumed to be the direct product of these coherent states. Through a Wigner transformation, the one-body phase space distribution function for $N$-distinguishable particles is given by:

$$f(r, p) = \sum_i \frac{1}{(\pi\hbar)^3} \exp \left[-\frac{(r - r_i)^2}{2\sigma_i^2} - \frac{2\sigma_i^2}{\hbar^2} (p - p_i)^2 \right]. \quad (2)$$

For identical fermions, the effects of the Pauli principle are discussed in Ref. 22. The approximative treatment of antisymmetrization is adopted in ImQMD model by means of the phase space occupation constraint method. The density and momentum distribution of a system respectively read

$$\rho(r) = \int f(r, p) d^3p = \sum_i \rho_i(r), \quad (3)$$
\[ g(p) = \int f(r, p) d^3r = \sum_i g_i(p) , \]  

respectively, where the sum runs over all particles in the system. \( \rho_i(r) \) and \( g_i(p) \) read:

\[ \rho_i(r) = \frac{1}{(2\pi\sigma_r^2)^{3/2}} \exp \left[ -\frac{(r - r_i)^2}{2\sigma_r^2} \right] , \]  

\[ g_i(p) = \frac{1}{(2\pi\sigma_p^2)^{3/2}} \exp \left[ -\frac{(p - p_i)^2}{2\sigma_p^2} \right] , \]  

where \( \sigma_r \) and \( \sigma_p \) are the widths of wave packets in coordinate and momentum space, respectively, and they satisfy the minimum uncertainty relation:

\[ \sigma_r \cdot \sigma_p = \frac{\hbar}{2} . \]  

The propagation of nucleons under the self-consistently generated mean field is governed by Hamiltonian equations of motion:

\[ \dot{r}_i = \frac{\partial H}{\partial p_i} , \quad \dot{p}_i = -\frac{\partial H}{\partial r_i} . \]  

The Hamiltonian \( H \) consists of the kinetic energy and effective interaction potential energy:

\[ H = T + U , \]  

\[ T = \sum_i \frac{p_i^2}{2m} . \]  

The effective interaction potential energy includes the nuclear local interaction potential energy and Coulomb interaction potential energy,

\[ U = U_{loc} + U_{Coul} . \]  

\( U_{loc} \) is obtained from the integration of the nuclear local interaction potential energy density functional. The nuclear local interaction potential energy density functional \( V_{loc}(\rho(r)) \) is taken as the same as that in Ref. 15, which reads

\[ V_{loc} = \frac{\alpha}{2} \rho^2 \rho_0 + \frac{\beta}{\gamma + 1} \rho^{\gamma + 1} \rho_0 + \frac{g_n}{2\rho_0} (\nabla \rho)^2 \]  

\[ + \frac{C_s}{2\rho_0} (\rho^2 - \kappa_s(\nabla \rho)^2) \delta + g_r \frac{\rho^{\eta + 1}}{\rho_0} . \]  

Here \( \rho, \rho_n, \rho_p \) are the nucleon, neutron, and proton density, respectively, and \( \delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) \) is the isospin asymmetry. By integrating \( V_{loc} \), we obtain the local interaction potential energy:

\[ U_{loc} = \frac{\alpha}{2} \sum_i \sum_{j \neq i} \frac{\rho_{ij}}{\rho_0} + \frac{\beta}{\gamma + 1} \sum_i \left( \sum_{j \neq i} \frac{\rho_{ij}}{\rho_0} \right)^{\gamma} + \frac{g_n}{2} \sum_i \sum_{j \neq i} f_{ij} \frac{\rho_{ij}}{\rho_0} (\nabla \rho)^2 \]
\[ + \frac{C_s}{2} \sum_i \sum_{j \neq i} t_{ij} \rho_{ij} \rho_0 (1 - \kappa_s f_{sij}) + g_\tau \sum_i \left( \sum_{j \neq i} \frac{\rho_{ij}}{\rho_0} \right)^\eta . \]  

(13)

where

\[ \rho_{ij} = \frac{1}{(4\pi\sigma^2)^{3/2}} \exp \left[ -\frac{(r_i - r_j)^2}{4\sigma^2} \right], \]  

(14)

\[ f_{sij} = \frac{3}{2\sigma^2} - \left( \frac{r_i - r_j}{2\sigma^2} \right)^2, \]  

(15)

and \( t_i = 1 \) and \(-1\) for proton and neutron, respectively.

The Coulomb energy is written as the sum of the direct and the exchange contribution, and the latter being taken into account in the Slater approximation

\[ U_{\text{coul}} = \frac{1}{2} \int \rho_p(r) \frac{e^2}{|r - r'|} \rho_p(r') dr dr' - e^2 \frac{3}{4} \left( \frac{3}{\pi} \right)^{1/3} \int \rho_p^{4/3} dR, \]  

(16)

where \( \rho_p \) is the density distribution of protons of the system.

Noting that the collision term and phase space occupation constraint can also readjust the momenta, but the former plays a very small role in low energy heavy-ion collisions and the latter only happens occasionally. The phase space occupation constraint method and the system-size-dependent wave-packet width are adopted as that in the previous version of ImQMD.\(^{14,15}\) The parameters used in this paper are given in Table 1, which are adopted in Refs. 16, 17.

### Table 1. The model parameters used in ImQMD calculations.

<table>
<thead>
<tr>
<th>( \alpha ) (MeV)</th>
<th>( \beta ) (MeV)</th>
<th>( \gamma )</th>
<th>( g_0 ) (MeV fm(^2))</th>
<th>( g_\tau ) (MeV)</th>
<th>( \eta )</th>
<th>( C_s ) (MeV)</th>
<th>( \kappa_s ) (fm(^2))</th>
<th>( \rho_0 ) (fm(^{-3}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-356</td>
<td>303</td>
<td>7/6</td>
<td>7.0</td>
<td>12.5</td>
<td>2/3</td>
<td>32.0</td>
<td>0.08</td>
<td>0.165</td>
</tr>
</tbody>
</table>
properly. Thus, in principle, the dissipation, diffusion and correlation effects are all included without introducing any freely adjusting parameter. The binding energies and root-mean-square for $^{197}$Au are required to be $7.92 \pm 0.05$ MeV/nucleon and $5.35 \pm 0.2$ fm, respectively. The stability of the pre-prepared initial nuclei is well checked. The pre-prepared nuclei are tested if they satisfy the following requirements, i.e., their binding energies and root-mean-square charge radii maintain proper values required by the properties of initial nuclei with a very small fluctuation and the bound nuclei evolve stably without spurious emission within 6000 fm/c.

Only those samples that satisfy the above requirements will be taken as the “initial nuclei” and are stored for usage in reaction simulations. By rotating these prepared projectile and target nuclei around their centers of mass by a Euler angle chosen randomly, we create more than 4000 bombarding events for each small impact parameter ($b = 0–6$ fm), 7000 bombarding events for $b = 7–9$ fm and at least 10000 events for each large impact parameter ($b = 10–12$ fm) at the energy of 15 A MeV, since the probability of ternary events decrease with increasing the impact parameter at this energy region. The distance from the projectile to the target at an initial time is taken to be 50 fm.

In the simulation, for comparison with the existing experimental data, we adopt the same criterion for selecting a class of ternary events satisfying nearly complete balance of mass numbers:

$$A_P + A_T - 70 \leq A_1 + A_2 + A_3 \leq A_P + A_T,$$

(17)
where $A_P + A_T$ is the total mass number. Through counting the number of $A_1$, $A_2$ and $A_3$ at each impact parameter $b$, the production cross sections for $A_1$, $A_2$ or $A_3$ are obtained with the expression

$$
\sigma(A_i) = 2\pi \int_0^{b_{\text{max}}} b P(A_i, b) \, db \\
\simeq \sum_{b=0}^{b_{\text{max}}} 2\pi b \Delta b \frac{N(A_i, b)}{N_0},
$$

where $P(A_i, b) = N(A_i, b)/N_0$ is the production probability of fragment $A_i$ with the impact parameter $b$. $N(A_i, b)$ denotes the number of $A_i$ producing at each impact parameter in ternary events, and $N_0$ denotes total ternary fission events. Here, $b_{\text{max}} = 12$ fm and $\Delta b = 1$ fm in this calculation.

With the aid of above criterion and formula, the mass distributions for each of three fragments in ternary fission are calculated for the first time with our microscopic transport theory. The results for selected ternary reactions of $^{197}$Au + $^{197}$Au at energy of 15 A MeV are shown in Fig. 1. The histograms denote the experimental data taken from Ref. 8, the lines with open circles are the calculation results with the ImQMD model. The experimental data and the calculation results are both normalized for comparing with 6000 and 35 mb in terms of the experimental counts and the calculated cross section. The results show us that the calculated results are in agreement with experimental data quite well. Here, it should be emphasized that these results are for the first time obtained by the microscopic transport model with the force parameter (Table 1) without any freely adjusting parameter. From the peak of mass number distribution of lightest fragments $A_3$ in the ternary fission being around 80 to 100, one can conclude that the most probable ternary events in the system $^{197}$Au + $^{197}$Au at 15 A MeV involve the formation of three comparable mass fragments. The ImQMD model can provide a reasonable description of the ternary fission events of very heavy system $^{197}$Au + $^{197}$Au.

4. Dynamical Mechanism of Ternary Modes for $^{197}$Au + $^{197}$Au

Now we turn to study the dynamical modes of ternary fission for system of $^{197}$Au + $^{197}$Au. There are two main possible modes: direct and sequential ternary fission, by which three comparable mass fragments can be produced. In a direct ternary event, two necks are formed and rupture almost simultaneously, and the three fragments centers are almost along a line. A typical snapshot of this type fission is drawn in Fig. 2. This figure clearly shows us the formation of composite system and ternary partition process of the system. In another mode, the designated sequential fission, a heavy fragment produced in binary division may have sufficient excitation energy to fission subsequently. This twice fission process is shown in Fig. 3. From this figure, we can see that with increasing the elongation of the system, the first neck is formed and then ruptured. During a very short time
Kinematic Correlation of the Ternary Fission for the System $^{197}$Au + $^{197}$Au

Fig. 2. (Color online) Snapshot of typical direct ternary fission of the $^{197}$Au + $^{197}$Au systems with impact parameter $b = 2$ fm and incident energy of 15 A MeV at different time. The open circles represent protons and solid ones neutrons.

the second neck again is formed and ruptured. The direct ternary fission is almost simultaneous process, while the sequential ternary fission is two-step fission process. Both dynamical modes for ternary fission can be observed in the simulation process of the ImQMD model.

5. Deviations from Viola Systematics

In order to reveal the nonstatistical feature of the ternary fission, we will look here at some kinematic correlations in dynamic process. We first construct the asymptotic relative velocities of the third fragment $A_3$ with respect to $A_1$ and $A_2$, $v_{\text{rel}}(1, 2) \equiv |v_{1,2} - v_3|$, respectively. Impact parameter is from $b = 1$ to 12 fm. We compare these quantities to the relative velocities from a pure Coulombian driven separation, in a hypothetical statistical fission process of a compound projectile-like or target-like system, as provided by the Viola systematics:\cite{24,25}:

$$v_{\text{viola}}(1, 2) = \sqrt{\frac{2}{M_{\text{red}}(1, 2)} \left( \frac{0.755 Z_1 Z_2}{A_{1,2}^{1/3}} + 7.3 \right)},$$

where $A_1$, $A_2$, $A_3$ and $Z_1$, $Z_2$, $Z_3$ are the mass and charge number of the heaviest, intermediate and the lightest fragments in the ternary fission, respectively.
$M_{\text{red}}(1,2)$ is the corresponding reduced mass of the sub-system for $(A_3 + A_1)$ or $(A_3 + A_2)$. We introduce the quantities $r$ ($r_1$), as the ratio between the observed sub-system $A_1 + A_3$ ($A_2 + A_3$) relative velocities and the one obtained from Viola systematics, i.e., $r = v_{\text{rel}}(1)/v_{\text{viola}}(1)$, ($r_1 = v_{\text{rel}}(2)/v_{\text{viola}}(2)$).

Figure 4 shows the probability distributions of $r$ and $r_1$ at different impact parameters. We can see that the most probable $r$ and $r_1$ at various impact parameters are larger than 1, and the deviation of the asymptotic relative velocities of $A_3$ with respect to $A_1$ ($A_2$) from Viola systematics increases with increasing of impact parameters. In fact, the $r$ ($r_1$) $> 1$ means that the fragment $A_3$ is dynamically separated from fragment $A_1$ ($A_2$). The stronger dynamic effect corresponds to the larger $r$ ($r_1$).

In Fig. 4, we also notice that there exists the second small peak in the $r$ and $r_1$ distributions when the impact parameters less than 7 fm. To understand these phenomena, we calculate the $r$ ($r_1$) distributions for each of two ternary fission modes. In Fig. 5, the $r$ ($r_1$) distributions for the direct and sequential ternary fission modes are shown by dashed and solid lines, respectively. One can see that the second peak in the $r$ ($r_1$) is mainly from the sequential ternary fission mode. The reason is that the first separation in sequential mode takes place much earlier than that for the direct ternary fission mode (see Figs. 2 and 3). Consequently, there is no enough time for the friction in sequential ternary fission to attenuate the
Fig. 4. Probability distributions of the deviations from Viola systematics of $A_3$ with respect to (a) $A_1(r)$ and (b) $A_2(r_1)$, respectively. Impact parameter is from $b = 1$ to 12 fm.
dynamical effects, and relative velocities will deviate much from statistical fission, thus forming the second peak in the \( r (r_1) \) distribution.

For further manifesting the dynamical effect, we represent the plot of \( r_1 \) against \( r \) at different impact parameters in Fig. 6. The dashed lines represent the loci of the projectile-like \((r = 1)\) and target-like \((r_1 = 1)\) fission events, respectively. Three groups of \( r - r_1 \) values are roughly found when the impact parameters are less than 8 fm, i.e., in the central and semi-central collision cases. One group with large \( r \) is on the \( r_1 = 1 \) line and another one with large \( r_1 \) on the \( r = 1 \) line, which mainly come from the sequential ternary fission already discussed above. The time of the second separation of this group plots is longer compared to that of the first one. During the longer time the friction will attenuate the dynamical effect, and relative velocities will deviate less from statistical fission. In the plane \( r - r_1 \) such events are located closer to the line \( r_1 = 1 \) or \( r = 1 \). The third group of \( r - r_1 \) values appears at the location that is simultaneously larger than one, this suggesting a weakened correlation of neck organizing fragment \( A_3 \) with both fragment \( A_1 \) and \( A_2 \), and
ruling out the statistical fission mechanism. Here we see very wide distributions of the \( r - r_1 \) correlation revealing a broad range of fragment velocities, typical of the instability evolutions in the neck region that will lead to large dynamical fluctuations on the properties of neck organizing fragment \( A_3 \). We can also see that the dots of the third group increase with the increasing impact parameter, and finally nearly all dots gather in the third group. This means that the dynamical effects increase with the rising impact parameter, till the largest parameter the dynamical effects dominate in ternary fission process.

6. Summary

In Summary, the ternary fission of the very heavy system \( ^{197}\text{Au} + ^{197}\text{Au} \) at energy of 15 A MeV has been investigated by using the ImQMD model. The experimental mass distributions of ternary fission fragments are reproduced well. The direct ternary and sequential ternary fission processes are studied by the time dependent snapshots of typical ternary events. In order to reveal the nonstatistical features of the ternary fission, we analyze the relative velocities deviations from Viola systematics. The probability distributions of the deviations from Viola systematics show us the nonstatistical feature of ternary fission in reaction \( ^{197}\text{Au} + ^{197}\text{Au} \) at 15 A MeV.
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